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ART
August OF 15th. 1746
PRACTICAL MEASURING,
BY THE
Sliding RULE,

hewing, how to meaſure round, ſquare or other Timber, Stone, Board, Glaſs, Paving, Painting, Wainſcot. Alſo GAUGING; with Inſtructions in Decimals, Mr. *Townley*'s Method of the Logarithms, and the Uſe of the Diagonal Scale applied to *Gunter*'s Chain.

By HENRY COGGESHALL, *Gent.*

Whereto is added, in a Short Method,
he Uſe of *Scamozzi*'s Lines, for finding the Lengths and Angles of Hips, Rafters, *&c.* at any Pitch, in Square, Bevelling or Tapering Frames.

By JOHN HAM.

And to render the Book more compleat, in this Edition is added,
COMPENDIUM of PRACTICAL GEOMETRY
nd the Principles of PLAIN TRIGONOMETRY, with
he Application thereof,

By G. THOMSON, *A. M.*

The FIFTH EDITION, carefully Reviſed.

LONDON:
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THE

7 - JUN 1926

PREFACE.

THE general Approbation of this Rule hath cauſed me to review the Book. I have explained ſome Things, and added others, which I thought uſeful.

Whether Mr. Townley's *Contrivance of the Indices be treated of any where but in Sir* Jonas Moore's *Mathematical Compendium, I could never learn; nevertheleſs, the Extraction of the Square and Cube Roots of Decimals, according to that Contrivance, being there wanting, I have given it you here; ſo that the Root of a Decimal, whoſe Logarithm hath ſuch an Index, is found with equal Eaſe as that of a whole Number.*

I have ſhewn how to find the Logarithm to any Number of ſix Places, in Mr. Wingate's Tabulæ Logarithmicæ, *(omitted in the Explanation of the Tables) by the Differences at the Bottom, and a Slip of this Rule, it ſupplying the Place of a Table of proportional Parts, which adds much to the Uſefulneſs of the ſaid Book, in giving the Logarithm of any Number, as far as a Million, with little Trouble.*

The Gauging of Tuns is here ſhewn according to the modern Practice of Inching. I have given you a near Way of meaſuring a Solid that tapereth ſtrait, as alſo of finding the total Content of the Conical Tun and Stand, from Mr. Everard, *whoſe Numbers for ten Inches Difference of Diameters, for the Parabola and Conoid, I have alſo inſerted. Thus far Mr.* Coggeſhall.

But in order to make this Treatiſe more acceptable, there are ſeveral Corrections, Emendations, and Additions made, particularly in ſhewing the Uſe of a new Three Foot Rule, which ſlides and ſhuts to a Foot, and which, for the Sake of the Reader's Peruſal, is neatly engraved upon *a Copper Plate by that accurate Mathematical Inſtrument-maker Mr.* Thomas Heath *in the* Strand; *for our Author's is entirely laid aſide, as not being ſo compleat; his making uſe of both Sides of the Rule in working Proportions, whereas this makes uſe but of one, which is not only a great Advantage in giving a Solution to all our Author's Propoſitions, but alſo reſerves the other Side of the Rule for* Scamozzi's *Lines; which are Lines known to moſt Workmen concerned in Buildings, to be of great Uſe in finding the Lengths and Angles of Hips and Rafters, and that by Inſpection, when the Roof is true Pitch; and if otherwiſe, is here ſufficiently illuſtrated by Schemes done from Copper Plates, there being an Example of a Frame that has the Gable End ſquare, and the Bevel hipt; and another where both Ends bevel, and the Sides run taper; in which laſt is ſhewn a new Method of determining the Quantity of the Angles, eſpecially of thoſe which relate to the Back of the Hips, ſo that they may anſwer both Sides and Ends of the Roof; and laſtly, an Example of finding the Lengths of Rafters for the Gable End of a Houſe that bevels or lies out of ſquare, and* *the fitting in of the Purloins, and finding their Lengths.*

Moreover, upon one Leg or Joint of the Rule there is a Table, with which, by Inſpection, you may know what a Load of Timber of 50 *Feet will amount to, at any Price from* 6 d. *to* 2 s. per *Foot.*

And if that which is here added prove uſeful, and render this Part of Building eaſy to Artificers in the Practice of their Buſineſs, it will be a Pleaſure to him, who upon all Occaſions, is glad of promoting real and uſeful Knowledge.

JOHN HAM.

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MAthematicks in Theory and Practice, *viz.* Arithmetick Vulgar and Decimal, with all the uſeful Rules, Merchants Accompts, or the true *Italian* Method of Book-keeping:

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By JOHN HAM,

Maſter of the Mathematical School at the Chapel in *Hatton-Garden*, *Holborn*: Which requiring his Attendance but three Days in a Week, *viz. Mondays*, *Wedneſdays*, and *Fridays*, the other three his Buſineſs lies altogether with Gentlemen at their own Houſes.

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* * *

THE

THE ART OF Practical Meaſuring, &c.

7 - JUN 1926

PART I.

ALtho' the way of Meaſuring by this Rule is ſo eaſy, that many, without the Art of Arithmetick, do underſtand and uſe it, as far as their particular Occaſions require; yet it is convenient for a Meaſurer to have ſome Skill therein, eſpecially in the five firſt Parts thereof.

There are Books of that Subject, written by divers, to be had at eaſy Rates, whereof only the later Authors treat of Decimal Arithmetick and Logarithms. By the former of theſe, the trouble-ſome Way of working with the vulgar Fractions is avoided; by the other, Multiplication and Diviſion are eaſily performed by Addition and Subtraction, and conſequently the moſt uſeful Parts of Arithmetick are rendered far more eaſy and expeditious.

Both theſe, ſince they firſt came into uſe, have been improved. The four Rules in Diviſion of Decimals, delivered by Mr. *Wingate* in his firſt Treatiſes of Arithmetick, are by Mr. *Kerſy* reduced to one, which follows after; and Mr. *Townley*'s Indices free the Operation by Decimals from Multiplicity of Rules, and give you the Rate or Value of them directly.

I have therefore here premiſed a ſhort Treatiſe of Decimal Arithmetick, and of the Nature and Uſe of the Logarithms; noting firſt, that the beſt way (in my Opinion) of reading Decimals, is, as you read whole Numbers, giving them the Name of the Value of the laſt Figure to the right Hand. To which purpoſe you may call them more eaſy, if inſtead of Tenths, Hundredths, Thouſandths, Ten-Thouſandths, *&c.* you ſay, Tenths, Cents, Milleſms, Decimilleſms, Centimilleſms, Millemilleſms, *&c.* as this .00165, I read thus, 165 Centimilleſms; and ſo of any other.

SECTION I.

Notation, Addition, Subtraction, Multiplication, and Diviſion in Decimals, alſo with whole or mixt Numbers.

A Decimal is the Numerator of a Fraction, whoſe Denominator is an Unit with one or more Cyphers; which Denominator is not expreſſed, it being known by the Numerator. For ſo many Places or Figures as are in the Numerator or Decimal,

mal, ſo many Cyphers are ſuppoſed to be adjoined to the Unit in the Denominator.

It ought to have a Point before it, as a Badge whereby it is known, being otherwiſe written as a whole Number: Therefore a Decimal of one place is Tenths, as this .2 is two Tenths; of two places is Hundredths, as this .02 is two Hundredths; of three is Thouſandths, as this .002, &c. the ſignificant Figures being put by the Cyphers into Places of leſs Value, and the Places decreaſing from Unit in a ten-fold Proportion.

Or thus. Call ſuch as have a ſignificant Figure next after the Point, as theſe, .3, .25, .732, &c. Decimals of the firſt Rank or Rate. Such as have one Cypher after the Point, as theſe .08, .0134, &c. Decimals of the ſecond Rank or Rate. Such as have two Cyphers after the Point, Decimals of the third Rate, &c. The Convenience whereof you will find afterwards.

The Work is the ſame as in the whole Numbers; yet take theſe Directions.

1. If you uſe a whole Number with mixt Numbers or Decimals, let it have always a Point after it.

2. In Addition and Subtraction, the Points prefixt to the Decimals muſt be ſet under one another, by which means the Units in the whole or mixt Numbers ſtand alſo under one another.

3. In Subtraction, although a Cypher at the right Hand of a Decimal is of no Value, (as theſe, .5, .50, .500, are no more than 5 Tenths, or one half) yet if the Decimals conſiſt not of an equal Number of Places, or if one of the Numbers be a whole Number, you muſt annex Cyphers, or ſuppoſe them annexed.

Examples of Addition.

22.75	19.2	.17	32.78	.34
9.6	4.68	.89	12.	.178
32.35	23.88	1.06	44.78	.84
				.7
				2.058

Examples of Subtraction.

22.75	19.2	107.	48.16
9.6	4.68	8.93	17.
13.15	14.52	98.07	31.16

Multiplication.

1. But in Multiplication, ſet them as if they were whole Numbers, and ſo multiply them: Cutting off from the Product found ſo many Figures to the right Hand, as there are Places of Decimals, both in the Multiplicand, and the Multiplier; ſo the Reſidue is the whole Part of the Product, and the Figures cut off, the Decimal.

2. If the Product hath not ſo many Places, as there are Places of Decimal Parts in both Numbers; ſupply the deficient Places with Cyphers prefixed to the left Hand.

Examples.

46.25	.87	564.	.0375	.76	.22
35.	.9	.25	.05	8.	.07
23125	.783	2820	.001875	6.08	1.54
13875		1128			
1618.75		141.00			

By theſe Examples you ſee how the whole Part of the Product is diſtinguiſhed from the Decimal Part thereof.

Diviſion.

In Diviſion it is ſomething harder to diſtinguiſh the whole Part of the Quotient from the Decimal Part thereof.

Firſt, Annex Cyphers to the Dividend, at pleaſure, or leave Space for them, that the Diviſion may be continued to a ſufficient Quotient. Then place the Diviſor under it according to the old way of Diviſion; but ſo as if they were both whole Numbers: Then obſerve well what Place or Degree of the Dividend ſtandeth over the Place of Units in the Diviſor; whether theſe Places be real, or only ſuppoſed; of the ſame Degree or Place is the firſt Figure in the Quotient. Which being once noted, you need not regard the Points nor Cyphers at either End of the Diviſor, any more; but continue the Diviſion, as if both were whole Numbers.

See here the Degrees of Numbers as they ſtand in their natural Order, which may be continued either Way from Unit.

Thouſ.	*Hund.*	*Tens.*	*Unit.*	*Tenths.*	*Hundredths.*	*Thouſandths.*
1000.	100.	10.	1.	.1	.01	.001, &c.

I have here ſet down ſome Examples, wherein you may ſee how the Diviſor is placed under the Dividend at the firſt Queſtion, and alſo the two firſt Figures in the Quotient.

Divid. 180.000 (30. &c. *Divid.* 1.75000 (.02, &c.
Diviſ. 5.875 *Diviſ.* 63.

Divid. 1.00000(.083,&c. *Divid.* .748000 (91. &c
Diviſ. 12. *Diviſ.* .0082

Divid. 9.72000 (21. &c.
Diviſ. .45

In the two laſt Examples; although there be neither Units Place in the Diviſor, nor (if there were) any Figure over it in the Dividend; yet by ſuppoſing the Places continued to the left hand, or ſupplying them with Cyphers, you will ſee that the firſt Place in the Quotient is the Place of Tens.

I account that way of Diviſion the beſt, in which, after (upon Examination by multiplying the Diviſor by the anſwering Figure from the left Hand toward the right) you have found the fit Figure to be put in the Quotient; you proceed, in your Diviſion to multiply the Diviſor by the anſwering Figure, beginning with the Figure in the Diviſor next the right Hand. If the Figure over it in the Dividend be not great enough to take the Product out of it; call it ſo many Tens, more than it is, as will make it great enough, but no more; and then ſubtract; ſetting the remaining Figure over it, and cancel the ſaid Figure: And for the Tens added, call the Product of the next Figure ſo many Units more than it is. Admit the Product 36 muſt be taken out of 2; call the 2, 42, and ſubtract. Suppoſe the next Product is 18; call it 22, *&c.* which Way you make feweſt Figures, and is no more Burthen to the Memory than ordinary Multiplication.

To multiply or divide any whole Number, mixt Number, or Decimal, by 10, 100, 1000, &c. *by removing the Point.*

To multiply: Remove the Point ſo many Places to the right Hand as there are Cyphers in the Multiplier: If Figures be wanting, ſupply them with

Cyphers, as here, 2.7 by 10 is 27. : .13 by 100, is 3 : 02 by 10, is .2.

To divide: Remove the Point ſo many Places o the left Hand, as there are Cyphers in the Di-iſor: If Places be wanting, ſupply them with Cyphers, as here, 27. by 10 is 2.7. : .13 by 100, is 0013 : .02 by 10, is .002.

SECT. II.

LOGARITHMS.

PUrpoſing to give you the Solution of ſome of the Queſtions in this Book by thoſe excellent Numbers the Logarithms; take theſe Directions or the better underſtanding the Nature and Uſe of hem.

They are artificial Numbers, fitted to the na-ural, for the Eaſe of Calculation; and are printed n Tables having two Columns. One hath the atural Number; againſt it in the other is his Logarithms: So that the Logarithm of a whole Number is eaſily found.

The Tables begin at 1, whoſe Logarithm is 0,00000; and reach commonly to 10,000; conſiſt-ng every one of 8 Figures, though (unleſs in great Numbers) we ſeldom uſe above ſix; (if the Figures eft out exceed 50, we put an Unit to the ſixth) o the Logarithms are annexed Differences; by the Help of which, and a Table of proportional Parts djoined, you are directed to find the Logarithm f any Number to 100,000. But theſe are but of Places.

Mr. *Wingate*, in his *Tabulæ Logarithmicæ*, hath he Logarithms to 100,000, with Differences alſo; whereby making a Proportion, (which is done ſpeedily by one Slip of this Rule) you have the Logarithms as far as 1,000,000, in a portable Volume

I

lume for the Pocket. A Book which I commend to any that delight in Arithmetick.

The firſt Figure, called the Index, (which is commonly ſeparated by a Point, better left out, except in the firſt hundred, as in the late printed Tables) ſhews how many Figures the anſwering Number, if whole, or the whole Part thereof, if it hath a Decimal annexed, conſiſteth of; which are always more by one than the Index. So 0. is the Index of one Figure, 1. of two Figures, 2. of three, 3. of four, *&c.*

Alſo according to the excellent Way of Mr. *Chriſtopher Townley*, cited by Sir *Jonas Moor* in his Mathematical Compendium, the Log. of a Decimal is the ſame, as if it were a whole Number, with this Direction for the Index.

If the Decimal be of the firſt Rate, the Index is 9; if of the ſecond Rate, the Index is 8; if of the third Rate, the Index is 7, *&c.* that is, the Index of the Logarithm of any Decimal, wants as many Units of Ten, as the left Hand ſignificant Figure is diſtant from Unity: Which, I hope, you will underſtand, if you obſerve this following Table.

Where you ſee, That in the perfect Numbers, the Index ſheweth the Number of Places in the whole Numbers, and in the whole Part of the mixt, being always leſs by one than the ſaid Places; but in Decimals it ſheweth the Rate, being the Complement thereof to Ten, not regarding the Number of Places.

Perf. Numb.	*Log.*
3536.	3·54851
353·6	2·54851
35·36	1·54851
3·536	0·54851
Decimals.	*Log.*
·3536	9·54851
·03536	8·54851
·003536	7·54851
·0003536	6·54851

If then you would have the Log. of any Number, find the Log. thereof in the Table, as if it were

ole ; and prefix the Index anſwering the Va-
e.

And having a Log. find the Number anſwer-
g in the Table, and by a Point fix the Value
cording to the Index.

find a Log. to a Number of ſix Places in the Tabulæ Logarithmicæ *by help of this Rule.*

Call the Difference at the Bottom the Tabular
ifferences: Having the Log. of the five firſt
gures, by the double Scale on your Rule, ſet 10
the Tabular Differences ; againſt your ſixth
gure is his proportional Part to be added to the
og. before found.

To find a Number of ſix Places anſwering a Log. given.

Find the Number of five Places anſwering the
og. in the Table, next leſs to the given Log. ſub-
act the ſaid Log. out of the given Log. call the
emainder the proper Difference ; then by the
uble Scale on your Rule ſet 10 to the Tabular Dif-
ence ; againſt the proper Difference on the ſecond,
your ſixth Figure on the firſt, to be annexed to
e five Figures before found.

Note, That you muſt uſe all the eight Figures
theſe Caſes.

Some Uſes of the Logarithms.

Whereas, before the aforeſaid Contrivance of the
dices by Mr. *Townley*, if one Number were per-
ſt, and the other a Decimal, there was a different
ule in every Operation for them ; but by the ſaid
ntrivance one is now ſufficient : I will give Ex-
ples only, in which one Number is a Decimal,
th theſe two Directions.

1. In the Log. which anſwereth the Queſtio (whether it be a Sum, Remainder, Half, &c.) the Index be ten, or above, neglect or cancel t ſaid Figure in the Place of Tens.

2. Where you are ordered to ſubtract a grea Log. out of a leſs; add ten to the Index of t leſs, and then ſubtract.

1. *Multiplication.*

Add the Logs. of the two, or more Numbers to be multiplied; the Sum is the Log. of the Product. So 12 multiplied by the Decimal .25, the Product is 3.

	12.	1.0791
By	.25	9.3979
	3.	0.4771

It may alſo be done where there are but tw by ſubtracting the Arithmetick Complement of t Log. of one of them out of the Log. of the other the Remainder is the Log. of the Product.

Which Arithmetical Complement is the Remainder of every Figure, (including the Index) to 9; except of the laſt ſignificant Figure to the right Hand, whoſe Remainder you muſt take to Ten. As in theſe three Examples.

Numb.		
2.	0.30103	*Log.*
.5	9.69897	*Ar. Comp*
Numb.		
80.	1.90309	*Log.*
.0125	8.09691	*Ar. Comp*
Numb.		
100.	2.00000	*Log.*
.01	8.00000	*Ar. Compl.*

2. *Diviſion.*

Subtract the Log. of the Diviſor out of the Log of the Dividend, (whether of the two be greater or leſs) the Remainder is the Log. of the Quotient. So 12 divided by the Decimal .25; the Quotient is 48.

	12.	1.0791
By	.25	9.3979
	48.	1.6812

It may alſo very conveniently be done, by adding Ar. Compl. of the Log. of the Diviſor to the g. of the Dividend; the Sum is the Log. of the otient, as followeth.

3. *The Rule of Three Direct.*

1. Add the Logarithms of the ſecond and third; m the Sum ſubtract the Log. of the firſt; the mainder is the Log. of the fourth.

2. A better Way: Add the Ar. Compl. of the g. of the firſt to the Logarithms of the ſecond third; the Sum is Log. of the fourth.

ample. If .25 give 16. hat ſhall 12. give?

Anſwer, 768.

Ar. Compl.	.25	0.60206
	16.	1.20412
	12.	1.07918
	768.	2.88536

But in the inverſe Rule: Add the Ar. Compl. the Log. of the third to the Logarithms of the t and ſecond; the Sum is the Log. of the fourth. us are reſolv'd the Queſtions wrought on the uble Scale.

But for thoſe in this Book, where there is a du-cate Proportion, as in Timber Meaſure and Gaug-, if the firſt and third Numbers be on the ſquare ne, there are general or fixt Logarithms belong- to the firſt Numbers; to which if you add the g. of the ſecond, and the Log. of the third twice, Sum of all four is the Log. of the fourth.

If the ſecond and fourth Numbers be on the are Line; to the Ar. Compl. of the Log of the t add the Log. of the third, and the Log. the ſecond twice, half the Sum is the Log. of fourth.

4. *The Square Root.*

Half the Log. of the Number given, is the full g. of the Square Root.

If the Number be a Decimal, add ten to the Index, and then halve it, as here.

.25	19.397
.5	9.698

5. *The Cube Root.*

The third Part of the Log. of the Number gi is the full Log. of the Cube Root.

If the Number be a Decimal, add twenty to the Index, and then divide by three, as here.

.25	29.397
.63	9.799

6. *To find a mean Proportional between two Numbers.*

Add their Logs. together: Half the Sum is Log. of the mean Proportional.

When one is a Decimal, if the Sum of the Indices be ten, (as here) or above; caſt away ten, and then halve it; if it be not ten, add ten to it, and then halve it.

12.	1.079
.25	9.397
	10.477
1.732	0.238

7. *To find two, or more mean Proportional between two Numbers.*

This, in Caſe of a Decimal, was ſomething plex'd, as you may ſee in Mr. *Wingate*'s Artif Arithmetick: It is now, by the aforeſaid Con vance of Mr. *Townley*, as eaſy, as it is uſeful.

Subtract the Log. of the leſs Number out of the Log. of the greater: The Remainder divide by a Number greater by one, than the Number of means ſought; as here by 4 for three means.

12.	1.079
.25	9.397
	1.681
	42

This Quotient added to the Log. of the leſs Number; the Sum is the Log. of the firſt Mean; to which adding again the ſaid Quotient, the Sum is the Log. of the ſecond Mean. And ſo forward for as many Means, as the Quotient was at firſt ordered for.

		Means.
		9.39794
		42031
1	.658	9.81825
		42031
2	1.732	0.23856
		42031
3	4.556	0.65887

8. *To find the Log. of a Vulgar Fraction.*

Subtract the Log. of the Denominator out of the Log. of the Numerator, the Remainder is the Log. of a Decimal equivalent to the ſaid vulgar Fraction.

$\frac{3}{4}$	0.47712
	0.60206
.75	9.87506

9. *To find the Log. of a Number with a Vulgar Fraction annex'd.*

Suppoſe it to be $12\frac{1}{4}$; change the Number into an improper Fraction, by multiplying the whole Number by the Denominator of the Fraction, and adding the Numerator to the Product, the Sum is the Numerator of the improper Fraction.

Then ſubtract the Log. of the Denominator out of the Log. of the Numerator, as before; the Remainder is the Log. of the ſaid Number, with a Decimal, equal to the ſaid vulgar Fraction, annex'd.

$\frac{49}{4}$	1.69020
	0.60206
12.25	1.08814

I have, as an Appendix to this Part, adjoined he uſual Decimal Tables, and compriſed them nto five: Yet the Uſe of them is as eaſy, as if hey were all ſingle.

The Integers, or Wholes, are ſet on the Top; nd the Parts follow in order, with their Decimals nnexed.

TABLE I.

A Table of Engliſh *Coin, a Pound Sterling, alſo Troy Weight, an Ounce* } *Integer.*

Shillings and Penny weight.	Decimals.	Pence with Farthings.	Decimals.	Grains.	*The Reſidue of the Table.*		
					Pence with Farthings.	Decimals.	Grains.
19	.95	3	.0489583		5	.0208333	10
18	.9	2	.0479166	23	3	.0197916	
17	.85	1	.046875		2	.01875	9
16	.8	11	.0458333	22	1	.0177083	
15	.75	3	.0447916		4	.0166666	8
14	.7	2	.04375	21	3	.015625	
13	.65	1	.0427083		2	.0145833	7
12	.6	10	.0416666	20	1	.0135416	
11	.55	3	.040625		3	.0125	6
10	.5	2	.0395833	19	3	.0114583	
9	.45	1	.0385416		2	.0104166	5
8	.4	9	.0375	18	1	.009375	
7	.35	3	.0364583		2	.0083333	4
6	.3	2	.0354166	17	3	.0072916	
5	.25	1	.034375		2	.00625	3
4	.2	8	.0333333	16	1	.0052083	
3	.15	3	.0322916		1	.0041666	2
2	.1	2	.03125	15	3	.003125	
1	.05	1	.0302083		2	.0020833	1
		7	.0291666	14	1	.0010416	½
		3	.028125	.	½	.0005208	½
		2	.0270833	13			
		1	.0260416				
		6	.025	12			
		3	.0239583				
		2	.0229160	11			
		1	.021875				

TABLE

TABLE II.

Averdupois *great Weight, One hundred at* 112l. *Integer.*

Quarters.	Decimals.
3	.75
2	.5
1	.25

Pounds.	Decimals.
27	.2410714
26	.2321428
25	.2232143
24	.2142857
23	.2053571
22	.1964286
21	.1875
20	.1785714
19	.1696428
18	.1607143
17	.1517857
16	.1428571
15	.1339286
14	.125
13	.1160714
12	.1071428
11	.0982143
10	.0892857
9	.0803571
8	.0714286
7	.0625
6	.0535741
5	.0446428
4	.0357143
3	.0267857
2	.0178571
1	.0089286

The Reſidue of the Table.

Ounces	Decimals.
15	.0083705
14	.0078126
13	.0072545
12	.0066964
11	.00[illegible]384
10	.0055803
9	.0050223
8	.0044643
7	.0039062
6	.0033482
5	.0027902
4	.0022321
3	.0016741
2	.0011161
1	.000558

Quarters.	Decimals.
3	.0004185
2	.000279
1	.0001395

TABLE III.

Averdupois *little Weight, one Pound* } *Integer.*
long Meaſure, one Yard or Ell,

Ounces	Decimals.	Qtrs with Nails.
15	.9375	3
14	.875	2
13	.8125	1
12	.75	3
11	.6875	3
10	.625	2
9	.5625	1
8	.5	2
7	.4375	3
6	.375	2
5	.3125	1
4	.25	1
3	.1875	3
2	.125	2
1	.0625	1

The Reſidue of the Table.

Drams	Decimals.	Qtrs of Nails.
15	.0585937	
14	.0546875	
13	.0507812	
12	.046875	3
11	.0429687	
10	.0390625	
9	.0351562	
8	.03125	2
7	.0273437	
6	.0234375	
5	.0195312	
4	.015625	1
3	.0117187	
2	.0078125	
1	.0039062	

Quarters.	Decimals.
3	.0029297
2	.0019531
1	.0009765

TABLE

TABLE IV.

Liquid Meaſure, one Gallon.
Dry Meaſure, one Quarter. } *Integer.*

Pints.	Decimals	Buſhels
7	.875	7
6	.75	6
5	.625	5
4	.5	4
3	.375	3
2	.25	2
1	.125	1

Quarters.	Decimals	Pecks.
3	.09375	3
2	.0625	2
1	.03125	1

Decimals	Quarters of a Peck.
.0234375	3
.015625	2
.0078125	1

Decimals	Pints.
.0058594	3
.0039063	2
.0019531	1

TABLE V.

Dozens, or Groſs
Time, one Year.
Long Meaſ. 1 Foot
Pence, one Shilling } *Integer.*

Dozens. Months.	Decimals	Inches Pence.
11	.9166667	11
10	.8333333	10
9	.75	9
8	.6666667	8
7	.5833333	7
6	.5	6
5	.4166667	5
4	.3333333	4
3	.25	3
2	.1666667	2
1	.0833333	1

Parts.	Decimals	Quarters & Farth.
11	.0763889	
10	.0694444	
9	.0625	3
8	.0555555	
7	.0486111	
6	.0416667	2
5	.0347222	
4	.0277778	
3	.0208333	1
2	.0138889	
1	.0069444	

Days belonging to the Table of Time.

Days.	Decimals.	Days.	Decimals.
30	.08219178	15	.0410959
29	.079452	14	.0383562
28	.0767123	13	.0356164
27	.0739726	12	.0328767
26	.0712329	11	.030137
25	.0684931	10	.0273972
24	.0657534	9	.0246575
23	.0630137	8	.0219178
22	.060274	7	.0191781
21	.0575342	6	.0164383
20	.0547945	5	.0136986
19	.0520548	4	.0109589
18	.0493151	3	.0082192
17	.0465753	2	.0054794
16	.0438356	1	.0027397

To bring Decimals into known Parts.

Multiply the Number of Parts in one Integer, and the Decimals together: From the Product cut off ſo many Figures to the right Hand as are in the Decimals (as you are directed in Multiplication of Decimals.) The Reſidue to the left Hand are the Parts ſought; and the Figures cut off are a Deci-mal of one of thoſe Parts, to be reduced the ſame Way into the next leſs Parts, if there be any, or i there be need. If nothing be left to the left Hand there is not one of thoſe Parts in that Decimal Therefore account it cut off, and proceed to fin the next leſs Parts, as before.

The making the foregoing Tables is by dividing the Numerator of the vulgar Fraction, which re-

preſent

eſents the Parts, by the Denominator; the Quo- ent is the Decimal. So $\frac{11}{20}$ being the vulgar raction of eleven Shillings or eleven Penny Weights; you divide 11 by 20, the Quotient .55 is the ecimal: So that half the Number of Shillings or enny Weights is the Decimal. Alſo $\frac{26}{960}$ being e vulgar Fraction of *6 d.* $\frac{1}{2}$, or of *26* Farthings; you divide *26* by *960*, the Quotient .0270830, *&c.* is the Decimal.

Yet you ſhall not need Diviſion for every De- imal; for ſome are found by halving the Integer r 1; and ſo continually: So are found the Deci- nal of one half, one quarter, one half quarter, *&c.* Some are found by halving a Decimal before ound: So half the Decimal of a Shilling, is the ecimal of Six-pence; half of that, the Decimal f Three-pence, *&c.* Alſo one third Part of the ecimal of a Shilling, is the Decimal of Four- ence; and the half of that, the Decimal of Two- ence, *&c.* and the double of it the Decimal of Eight-pence. Likewiſe the Sum of two Decimals, is the Decimal of the Sum of the two Fractions, whoſe Decimals they are; and the Difference is the Decimal of their Difference.

Some of theſe are of one Place, and ſome of more: Few Tables have them to above ſeven; and moſt ordinary Queſtions may be reſolved to a ſufficient Exactneſs, if you uſe but four; remembering the Direction above given, *viz.* If the firſt Figure of thoſe left out exceed 5, to add a Unit to the laſt of thoſe you retain.

If the Anſwer of a Queſtion be in Money, three Places of Decimals give it to near a Farthing, as is ſhewn after *Part* 4. *Prop.* 5.

Now for the Uſe of them in a Queſtion or two.

1. At 5*s.* 3*d.* $\frac{1}{2}$ the Ounce; what coſt 7 Ounces, 3 Penny Weight, and 19 Grains?

Having added the Decimals of the Parts, th Queſtion will ſtand thus :

ou. *l.* *ou.* *l.*
1 : 0.2645833 : : 7.1895833 : 1.902

The Product or Anſwer is 1 *l.* 9022, &c. Which is 1 *l.* 18 *s.* 0 *d.* 2 *f.* near.

If you leave out the three laſt Figures in each Decimal, with the Condition above-mentioned, the Numbers are

ou. *l.* *ou.*
1 : 0.2646 : : 7.1896,

And the Anſwer is 1 *l.* 9023, &c. differing from the other inconſiderably.

2. To compute ſimple Intereſt for any Sum, Rate, and Time. Having put the Parts, if there be any, into their Decimals ; multiply the Principal and the Rate ; from the Product cut off the due Decimal, if any, and two Places more for the Diviſion by 100 : This Product ſo ordered is the Intereſt due for one Year ; which if you multiply by the Time, (be it more or leſs than a Year) the Product (the due Decimal cut off) is the Intereſt for that Time.

Examp. 1. What is the ſimple Intereſt of 132 *l.* 7 *s.* 6 *d.* for 2 *y.* 3 *m.* 22 *d.* at 6 *l.* in the Hundred ?

The Decimal of 7 *s.* 6 *d.* is .375 ; which being annexed to the whole Pounds, the Principal will be 132 *l.* 375 ; which multiply'd by 6, and the Product ordered as directed, it will be 7.9425, or 7 *l.* 18 *s.* 10 *d.* 1 *f.* near, for the Intereſt for one Year. But that being not the Sum ſought, multiply the ſaid 7.9425 and the Time, *viz.* 2 *y.* 3103, the Product 18.3493 is the Intereſt ſought, *viz.* 18 *l.* 6 *s.* 11 *d.* 3 *f.*

Ex. 2. What is the Intereſt of the ſaid Sum for
o Months and ten Days at the ſame Rate? Mul-
ly the ſaid 7.9425 by .1941 the Decimal of the
me, the Product 1 *l.* 5416, or 1 *l.* 10 *s.* 10 *d.* is
e Intereſt ſought.

But the great Convenience of Decimals is,
at their Logs. are ſo eaſily found; as is already
ewn in this ſecond Section. So that by the
bulæ Logarithmicæ mentioned in the afore-
ed Place, any Queſtion, whoſe Numbers (whe-
er whole, mixt, or Decimals) exceed not ſix
aces, may be ſpeedily reſolved: Mr. *Town-*
's Indices of the Decimals freeing us from
rplexity of different Rules. As in the two laſt
amples.

To the Arith. Compl.
the Log. of 100, *viz.*
000000 add the Log.
the Principal and of
e Rate; the Sum is
e Log. of the Inte-
t for one Year. To
ich Log. if you add
e Log. of the Time,
is Sum ſhall be the
g. of the Intereſt for
e Time.

Or without ſeeking
e Intereſt for one Year:
the ſaid Ar. Compl.
d the Logs. of the
incipal, Rate, and Time, the Sum ſhall be the
g. of the Intereſt demanded, as in the ſecond
ample.

Example 1.

100. *Ar.Com.*	8.
132.375	2.1218052
6.	0.7781512
7.9425	0.8999571
2.3103	0.3636683
18.3495	1.2636254

Example 2.

100. *Ar.Com.*	8.
132.375	2.1218059
6.	0.7781512
.1941	9.2880255
1.5416	0.1879826

3. Compound Intereſt for any Principal, Rate,
d Time by the Logarithms.

In this Propoſition the Excellency of thoſe Nu bers appear ; ſuch Queſtions being reſolved b them with great Eaſe and Speed ; but by natur Arithmetick not without conſiderable Time an Trouble.

Deduct the Log. of 100 from the Log. of 10 and the Rate added together, as 105, 106, & The Difference multiply by the Time: From th Product cut off the Decimal, if there be any : Th Remainder add to the Logarithm of the Principal the Sum is the Logarithm of the Principal an Intereſt required.

Example.

Let the Principal, Rate, and Time be as in th former of the two laſt Queſtions. Purſuing the Rule, as you ſee in the Margin ; the Sum of the principal and compound Intereſt is 151 *l.* 9 *s.*

The Difference		25305
The Time		2.310
The Product		584639.897
	132.375	2.121805
		58463
	151.45	2.180269

It ſeems by this, that the Intereſt of 100 *l.* a 6 *l. per Cent.* by the Year, is not fully amounte to 3 *l.* in ſix Months; for if you multiply th aforeſaid Difference by 5, the Decimal of ſi Months ; and, having cut off one Place, add th Reſidue to the Log. of 100, the Sum will b 2.0126529 ; which is the Log. of 102.956, that i 102 *l.* 19 *s.* 1 *d.* 1 *f.*

I will add two or three Examples more, which I hope, will be ſufficient.

1. What is the Value of 28 Ounces, ſix Penny Weights and 15 Grains of Gold, at 3 *l.* 3 *s.* 6 *d*

Ounce? Annexing the Decimals to the Inte-
s, the Numbers ſtand thus:

ou. *l.* *ou.* *l.*
1 : 3.175 :: 28.33125 : 89.952

l. *s.* *d.* *f.*	3.175	0.5017437
cit, 89 . 19 . 00 . 2	28.33125	1.4522657
	89.952	1.9540094

ou. *p.* *gr.* *l.* *s.* *d.*
. If 4 . 9 . 12 of Gold coſt 14 . 10 . 9;
hat is that the Ounce?

ou. *l.* *ou.* *l.*
e Numbers are 4.475 : 14.51875 :: 1 : 3.2444

l. *s.* *d.* *f.*	4.475	*Ar. Compl.*	9.3492070
cit, 3 . 04 . 10 . 2.	14.51875		1.1619291
	3.2444		.5111361

. At 6 *s.* 3 *d.* the Ounce; how much Silver
te will 5 *l.* 3 *s.* 6 *d.* buy?

l. *ou.* *l.* *ou.*
The Numbers are 0.3125 : 1 :: 5.175 : 16.56

ou. *p.* *gr.*	.3125	*Ar. Compl.*	0.50515
cit, 16. 11. 05 near.	5.175		0.71391
	16.56		1.21906

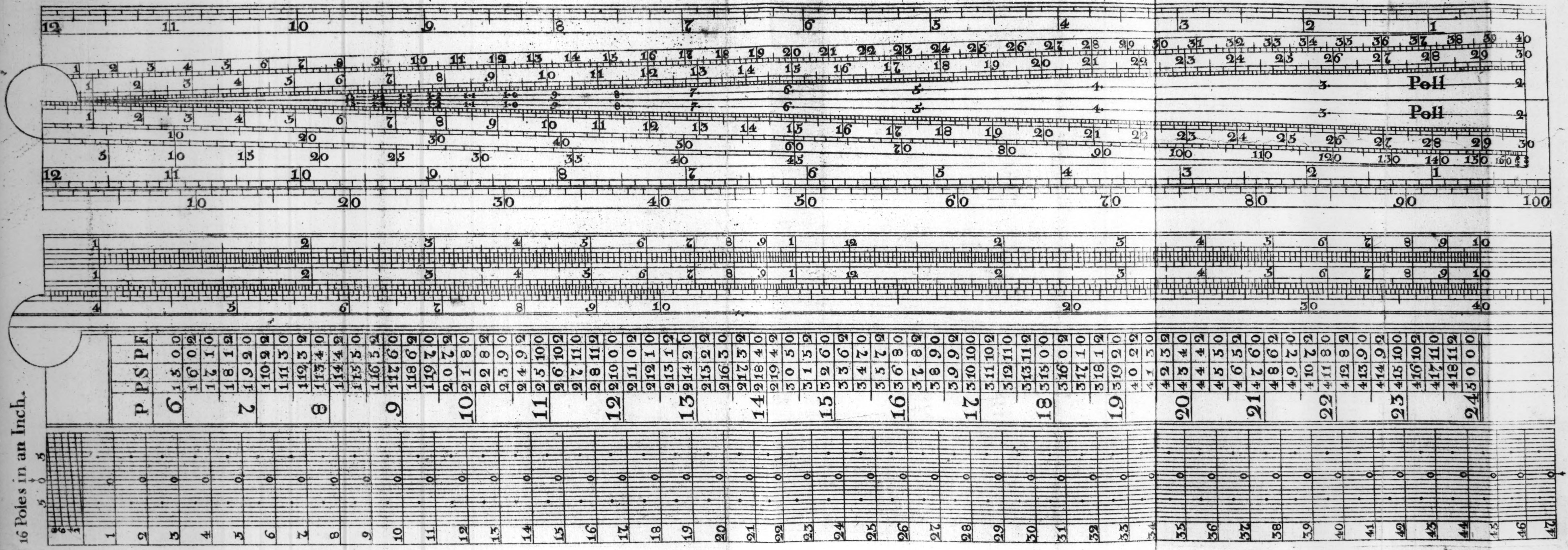
I have taken but ſix Figures in this laſt Example.
I had uſed no more in the other, the Difference
uld have been little or inconſiderable; as you
y find if you pleaſe to give yourſelf that ſmall
ouble.

Theſe thus premiſed, I ſhall come next to
Deſcription and Uſes of the Rule in ſeveral M
ſures ; wherein I ſhall uſe theſe vulgar Fractio
viz $\frac{1}{4}$ one Quarter, $\frac{1}{2}$ one Half, $\frac{3}{4}$ three Quarte
The Decimals belonging to theſe, as they are i
mediate Parts of the whole, are .25 for a Quart
.5 for an half, and .75 for three Quarters.

But if they be Parts of Parts, other Decim
belong to them, as you ſee in the Tables.

◆

TH



16 Poles in an Inch.

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THE ART OF Practical Meaſuring, &c.

PART II.

The Deſcription of the Rule.

THIS Rule is different from that deſcribed by Mr. *Coggeſhall*, and which was all along made uſe of in the former Editions, but now is laid aſide, as not being ſo compleat; for his made uſe of both Sides of he Rule in working Proportions, whereas this akes uſe but of one; which is not only a great dvantage in the Performance of the Operations, ut alſo reſerves the other Side for the Lines made uſe of in *Scamozzi*'s Architecture, for the ready finding the Lengths and Angles of Rafters, &c. Of which more in its proper Place.

Therefore in *Fig.* I. to give a Deſcription of that Side of the Rule, which gives a Solution to all Mr. *Coggeſhall*'s Propoſitions, there is a Piece made to ſlide in and out, having on each Edge thereof a Line of Numbers in two Lengths, *viz.* from 1 at the Beginning to 1 in the Middle, which is eſteemed one Length*; and from 1 in the Middle to 10 at the End, the other. Radius.

There are alſo adjoining to this Piece two Lines, on one Side, a Line of Numbers in all Reſpects like thoſe juſt deſcribed; and on the other, a Line figured 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, called the Square Line, and when the Meaſure of round Timber is concerned, the Girt Line.

The Lines of Numbers on the ſliding Piece, and that adjoining thereto, are divided between 1 and 2 into 10 Parts, and each Tenth into 5 Parts; between 2 and 3 into 10 Parts, and each Tenth into 2 Parts; from 3 to 4, and from 4 to 5 it's the ſame; from 5 to 6 into 10 Parts only; and ſo on to 1 in the Middle. The ſecond Length or Radius is divided exactly like the firſt.

The Girt Line from 4 to 10 has each Tenth divided into 2 Parts, and from 10 to 40 each Whol into 4 Parts; alſo the Diviſions 37, 38, and 3 with their Halfs and Quarters, are put on befor 4 at the Beginning of this Line, and cut, as i their proper Places, towards 40 at the other En of the Line; from whence the Diviſions are carried on to 4 , each Diviſion between 40 and 4 being divided into two Parts.

There is on the ſame Side of the Rule but u on the other Leg, a Table, whereby you m know, by Inſpection, what a Load of Timb

wi

* or Radius

vill amount to at any Price between 6 *d.* and 2 *s.* *per* Foot, reckoning 50 Feet to the Load.

Example 1.

At 9 *d. per* Foot, a Load of Timber will come o 1 *l.* 17 *s.* 6 *d.*

Example 2.

At 10 *d.* ¼ *per* Foot, a Load will amount to *l.* 2 *s.* 8 *d.* ½.

When the Rule is opened, ſo as to become one ght Line, you will find that the Back or outer dge thereof is decimally divided; that is, each oot is divided into 10 equal Parts, and each Part to 10 more, by which means the Rule is divided to 200 equal Parts; and with this Line I mean imenſions ſhould be taken, when I ſpeak of Foot eaſure.

Next to this outer Edge, on the other Side of e Rule, where *Scamozzi*'s Lines are, is a Line Inches decimally divided, and conſequently ch Inch is divided into 10 equal Parts, which is r Gauging.

Note, on the Back-ſide of the ſliding Piece is a ot Rule, whoſe Inches are divided into Halves, uarters, and half Quarters.

That Line of Numbers upon the ſliding Rule, ith that Line of Numbers which is next to it the fixed one, is in this Book called the double ale.

SECTION I.

Of the Line of Numbers, commonly called Gunter's *Line.*

The proper Numeration of this Line I account, 1 at the Beginning; and ſo by 10 in the Middle, to 100 at the End.

But for the better underſtanding this Line, ſee here the Degrees of Number from Unit on either Side, as they ſtand on the Line; that is, increaſing from left Hand to right.

Thouſ. pts.	*Hund. pts.*	*Ten. pts.*	*Unit.*	*Ten. pts.*	*Hund.*	*Thouſ*
.001	.01	.1	1.	10.	100.	1000

Where you ſee, how they increaſe on one Side from Unit, and on the other Side decreaſe from the ſame in a tenfold Proportion. So that if you ſet any one of them at the Beginning, the two next following ſhall be, one the Middle, the other the End. As if you call 1 at the Beginning one tenth, the Middle ſhall be 1, and the End 10. If you call one at the Beginning 10, the Middle ſhall be 100, and the End 1000. But if it be not otherwiſe limited, account it, as I ſaid before, 1, 10, 100.

On that Line of Numbers which is on the moveable Rule, at theſe Numbers following may be Pricks, for the more ready finding them; they being firſt Numbers or Centers, as they commonly call them.

At 9 for Yard Meaſure.

At 12 four Points thus . : . for Plank or Board and Glaſs.

At 144 alſo for Glaſs.

At 160 for Land Meaſure.

A

At 272, 25, or 272 for Rod Meaſure of Brick-work at 1 ½ Brick thick; for the Decimal .25 equal ¼ may be omitted without conſiderable Error.

At 204 for two Bricks thick, and other Points or Cuts for other Thickneſſes, if deſired. The finding whereof is ſhewed after.

At 282 for Ale Gallon Meaſure in ſquare and oblong Veſſels. And other may be ſupplied, as any hath occaſion for them.

SECT. II.

Of the Square or Girt Line.

THIS is no more but one whole Length of the Line of Numbers, but at a double Radius, it being exactly equal to the Lines of Numbers on his Rule, which are in two Lengths.

In the Numeration of it, when occaſion requires, you muſt account 10, 20, 30, to be 1, 2, ; as alſo 4, 5, 6, 7, &c. to be 40, 50, 60, 70, &c.

At 12 on this Line let there be four Pricks, as t 12 on the Line of Numbers: At 17.15, the Vine Point, marked W. At 18.95, the Ale oint, marked A. Theſe are put on by the Workman.

At 10.635 may be a Point like the Gauge Points, or finding the true Content of a round Solid by he Girt.

At 13.54 ſuch another, for finding the Content f a Cylinder by the Diameter.

At 41.57 another, to ſhew how many Inches in ength make a Foot Solid at any Girt or Square ot exceeding 40 Inches.

Theſe may be put on thus:

For the firſt: Set 12 on the ſquare Line to 14 on the Line of Numbers; againſt 11 on the Line of Numbers mark this Point.

The ſecond: Set 12 on the ſquare Line to 11 on the Line of Numbers; againſt 14 on the Line of Numbers mark this Point.

The third: Set 12 on the ſquare Line to 1 at the Beginning of the Line of Numbers; againſt 12 on the Line of Numbers mark this Point.

Theſe, or any other mentioned hereafter, cut with a ſharp-pointed Penknife in two Places, as the Gauge Points, and ſtrike in with your Finger ſome Sallow Coal fine ground with Linſeed Oil, and then wipe the Rule clean.

Both theſe Lines are put on from the Logarithms.

SECT. III.

The general Uſe of double Scales.

IT is chiefly for the working the Rule of Thre or having three Numbers given, to find fourth Proportional, it including Multiplicatio and Diviſion; for there is no other Differenc than that in theſe two an Unit is one of th three.

To find this fourth: Set the firſt Number o the ſliding Rule to the ſecond on the Line o Numbers on the fixed Rule; againſt the thir Number on the firſt of theſe Rules, is the fourt on the ſecond Rule.

Example. If 2 give 3, what ſhall *6* give? S 2 on the firſt to 3 on the ſecond; againſt *6* on th firſt is 9 on the ſecond.

Wherefore when I ſay, ſet 2 to 3, againſt *6* 9, I mean as in the above-ſet Example, tho' ame not the firſt or ſecond Rule, yet you may ote that the moveable Rule is for the moſt Part he firſt.

So in Multiplication, 1 being the firſt Number, et one to either of the two Numbers to be mul- iplied, (beſt to the neareſt) againſt the other is he Product.

In Diviſion, the Diviſor being the firſt Number, et it either to one, or to the Dividend, againſt the ther is the Quotient. Examples of both you will eet with after.

By this the Arithmetick of the Rule is eaſily nderſtood, the firſt Numbers being Diviſors; nly where the ſquare Line is uſed, the Numbers n the ſquare Line muſt be ſquared or multiplied y themſelves, and their Squares uſed in every eſpect as if they were the Numbers themſelves, s you will after ſee.

SECT. IV.

To ſet the ſquare Line to his Squares, and thereby to ſquare a Number not exceeding 100, *and to find the ſquare Root of a Number not exceeding* 10,000.

1. SET 10 on the ſquare Line to 1 at the Beginning of the Line of Numbers, both which, if you account 1, you have on the Line of Numbers the Squares from 1 to 20. Alſo at this ſet as is moſt convenient to take the Squares from 100 to 2000, *viz.* by accounting 10 on the ſquare Line 10, and 1 at the Beginning 100.

2. Set 10 on the ſquare Line to one in the Middle, accounting 10 on the ſquare Line 10, and 1 in the Middle 100, you have the Squares on the Line of Numbers from 16 to 1000; and of this you will have moſt Uſe.

3. Set 10 on the ſquare Line to 10 at the End, accounting 10 on the ſquare Line 100, and 10 at the End 10,000; ſo have you on the Line of Numbers the Squares from 1400 to 10,000; and againſt all the Squares, at every Set, his own Root upon the Square or Girt Line.

Example. I would know what is the ſquare Root of 380?

Set 10 on the ſquare Line to 1 in the Middle, (according to the ſecond Direction) againſt 380 on the Line of Numbers is 19.5 near.

SECT. V.

A Direction concerning the Shortneſs of the Lines on the Rule.

1. IN working theſe Proportions on the double Scale, if you uſe half of either the ſecond or third Number inſtead of the whole, you will have half the Content; if you uſe the Half of both, you have only a Quarter of the Content. *Note,* The firſt Number muſt always remain whole.

2. On the ſquare Line: If you uſe half that Number that is on the Line of Numbers, commonly the ſecond, (being ordinarily Lengths or Depths) you have half the Content.

If half of that on the ſquare Line, commonly the third, (being ordinarily Sides of Squares, or Diameters) you have a Quarter of the Content; if you halve them both, you have only one Eighth of the Content.

3. In finding a mean Proportional; half the Extremes give half the Means, and a Quarter gives a Quarter. Theſe may be more exactly defined, and multiplied accordingly.

This Direction alſo may be uſeful to you in orking by the ſquare Line, when at any Time e third Number ſtandeth beyond the Line of umbers; and removing the firſt to the ſecond in e other Length, ſets it farther off. For then,

1. If the firſt be a fix'd Number on the ſquare ine, as 12, and the Gauge Points, &c. ſet it to alf the ſecond, and double the Content.

2. If the firſt and ſecond be fix'd, as in finding he ſuperficial Content by the Diameter, uſe half he third Number being on the ſquare Line, and uadruple (or multiply by 4) the Content.

These Products are the 4th Numbers ſought.

THE ART OF Practical Meaſuring, &c.

PART III.

PROB. I.

To meaſure round Timber the common Way.

MEaſure the Length in Feet and half Feet, and (if the Cuſtom or Agreement be ſo) in Quarters; then back again half way, where girt the Tree with a ſmall Cord or Chalk Line, double this Line twice very even. This fourth Part of the Circuit, (which in this Treatiſe I call the Girt) meaſure in Inches,

nches, halves, and quarters of Inches. And this bſerve, that the Lengths be given in Feet, the irts and Sides of Squares in Inches.

So have you three Numbers given, *viz.* 12 always the firſt, the Length always the ſecond, and he Girt or Side of the Square the third.

To come now to the Rule: Set 12 on the irt Line to the Length on the Line of Numbers; gainſt the Girt on the Girt Line is the Content n the Line of Numbers. And this is the general ule.

Now there being two Caſes, one when at the irſt Set the Girt is againſt ſome Part of the Line f Numbers, the other when it is not, ſo that 12 uſt be removed, I will give you ſeveral Examles of both, obſerving that the vulgar Fractions efore-mentioned, as alſo all Decimals, always folow the Number they belong to before the Name hereof.

CASE I.

EXAMPLES.

1. A Tree is 20 Foot long, and 15 Inches girt, et 12 to 20, againſt 15 is 31 ¼ Foot, or 31 Foot nd a Quarter.

2. A Length is 8 ½ Foot, the Girt is 35 ¾ Inches, et 12 to 8 ½, againſt 35 ¾ is 75 Foot and almoſt n half.

3. A Length is 15 Foot, the Girt 42 ½ Inches, et 12 to 15, againſt 42 ½ is 188 Foot.

4. A Rail is 15 Foot long, the Girt 3 Inches, et 12 to 15, againſt 3 is 9 Tenths of a Foot, and nore.

5. A Length is 9 ½ Foot, the Girt 39 ¾ Inches, et 12 to 9 ½, againſt 39 ¾ at the Beginning of the irt Line is 104 Foot.

6. The Length is 0.62 Foot, the Girt 35 Inche ſet 12 to the Decimal .62 in the firſt Length againſt 35 is 5 ¼ Foot, which may ſerve for a ſhor Cut of a Tree.

If this Length had been propounded 7 ½ Inche it muſt have been turned into Foot Meaſure thus On the double Scale ſet 12 on the ſliding Rule, t 100 on the Line of Numbers on the fixed Rul againſt the Length in Inches is the Length in Foo Meaſure, equal to .625 of a Foot; but if it lie before you, meaſure it by the Line of Foot Mea ſure on the Rule.

CASE II.

If at the firſt Set the Girt is beyond the Line o Numbers, remove 12 to the Length in the othe Length thereof. Which Caſe may alſo happen i Gauging, &c.

Example 1. The Length is 18 Foot, the Gi 31 Inches; ſet 12 to 18 in the firſt Length, again 31 is 120 Foot.

2. A Rail is 15 Foot long, the Girt 3 ½ Inches ſet 12 to 15 in the firſt Length, againſt 3 ½ is Foot, and a little more, *viz.* 1.27 Foot.

3. A Wrong is 6 ½ Foot long, and 4 ¼ in Girt ſet 12 to 6 ½ in the ſecond Length, againſt 4 ¼ above eight Tenths of a Foot. Theſe Exampl may be ſufficient.

Note 1.

If you would find the Content of a great Pie of Timber immediately in Loads, at 40 Foot t the Load, uſe half the Girt inſtead of the whol *Example.* A Length is 15 Foot, the Girt 42 Inches; ſet 12 to 15, againſt 21 ¼ is 47, where the 4 is 4 Loads, and the 7 is 28 Foot.

If by this Way you meaſure Timber, whoſe Girt is above 40 Inches, as alſo the Piece in *Caſe* 1. *Ex*. 5. which, without the ſaid Subdiviſions, and placing 38 and 39 before 4 at the Beginning, are not reſolved by the general Rule.

But if you would have the Content of theſe Pieces in Feet, multiply the Content found by 4, the Square of 2, by which you divided your Girt; ſo 47 multiplied by 4 is 188 Foot.

Note 2.

To what Length ſoever you ſet 12, 17 will ſtand to the Double thereof, 8 ½ to Half thereof, both a little over; alſo 24 will ſtand to the Quadruple thereof, and 6 to a Quarter thereof exactly; and the ſame Proportion the Content bears to the Length at any of theſe Girts, *viz*. at 17 Inches Girt, the Content is double to the Length; at 8 ½ Inches Girt, the Content is but half the Length, &c.

Note 3.

If you would find theſe Contents by natural Arithmetick, ſeeing 12 and the Girt, *viz*. the irſt and third Numbers are on the ſquare Line, ccording to a Hint given in *Sect*. 3. of *Part* 2. ultiply the Square of the Girt by the Length, nd divide the Product by 144, the Square of 12, hich is your conſtant Diviſor, the Quotient is the ontent.

So in *Ex*. 1. *Caſe* 1. the Square of 15, *viz*. 225, ultiplied by 20, and the Product 4500, divided y 144, the Square of 12, the Quotient is 31.25, he Content.

By the Logs. to this general Log. 7.84164, add the Log. of the Length and the Log. of the Girt twice; the Sum is, The Log. of the Content.

Gen. Log.	7.84164
20	1.30103
15	1.17609 1.17609
31.25	1.49485

PROP. II.

True Meaſure of round Timber or Stone by the Girt.

BEcauſe this common Way of meaſuring round Timber giveth not a true Content, but always too little, (tho' it ſtill be generally uſed) I have given you a Point, and ſhewn how you may pu it on the Rule, which ſetting to the Length inſtead of 12, the Girt ſhall point you out a true Content accounting it a Cylinder, as the ſaid common Way alſo doth.

Example. Let the Length be 10 Foot, the Gir 15 Inches; ſet the ſaid Point mark'd at 10, 635 on the Square or Girt Line (which you may call th true Point) to 10; againſt 15 you have 20 Foo leſs by about one Tenth; whereas the commo Way giveth but 15 Foot, and a little above a Half.

The general Logarithm anſwering this Point i 7.94652, to be uſed as before.

Thus far by the Length and Girt: I ſhall onl add, that the common Meaſure is to the true a 11 to 14; ſo that if you ſet 11 on the doubl Scale to any Number of Feet or Loads meaſure the common Way, 14 ſhall point to the true Con tent of the ſame; and if you ſet 14 to any tru Content, againſt 11 is the Content the commo Way.

PROP. III.

Having the Length of a Cylinder in Feet, and the Diameter in Inches, to find the Content in Feet.

SET the Point 13.54 to the Length, againſt the Diameter is the Content. *Example.* Let the Length be 10 Foot, and the Diameter 20 Inches, ſet 13.54 to 10, againſt 20 is 21.82 Foot; the general Log. is 7.73676, to be uſed as before.

PROP. IV.

Having the Length of a ſquare Solid in Feet, and the Side of the Square in Inches, to find the Content in Feet.

SET 12 to the Length, againſt the Side of the Square is the Content. The Caſes are as in ound Timber; the Examples alſo will ſerve, acounting the Girts to be Sides of Squares.

PROP. V.

To find a mean Proportional between two Numbers.

SET the greater of the two Numbers on the ſquare Line to the ſame on the Line of Numers, againſt the leſs on the Line of Numbers is e mean Proportional on the ſquare Line.

Or ſet the leſs on the ſquare Line to the ſame n the Line of Numbers, againſt the greater on e Line of Numbers is the mean Proportional on e ſquare Line. One of theſe will not fail. Exmples follow in the next.

PROP. VI.

Unequal ſquared Solids.

MEaſure the Length in Feet, the Breadth and Depth in Inches; then find a mean Proportional between the Breadth and Depth, as is taught above; and it will be the Side of a Square equal to the Baſe or End; which having found, meaſure the Piece as ſquare Timber.

Example 1. In Timber, whoſe Length let be 10 Foot, the Breadth 21 Inches, the Depth 8½ Inches; ſet 21 to 21, againſt 8½ on the Line of Numbers is 13.36, or 13 a Quarter and half Quarter near; or ſet 8½ to 8½, againſt 21 on the Line of Numbers is the ſaid 13.36; then ſetting 12 to 10, againſt 13.36 is 12.4 Foot.

Example 2. In Stone, which let be 6.35 Foot long, 36½ Inches broad; and 5.7 Inches deep; ſet 36½ to the ſame, againſt 5.7 on the Line of Numbers is 14, and ſomething ſhort of an half; then ſet 12 to 6.35, againſt this Mean is 9.2 Foot near.

This Mean, in Caſe of a Fraction, ſhall give you no Trouble; for if with a Pencil, Chalk, or any thing that may be wiped out without Damage to your Rule, (let it not be Ink) you make a fine Mark on the ſquare Line at this Mean, and then ſet 12 to the Length, this Mark, without defining it, ſhall point out your Content.

PROP.

PROP. VII.

Solids of a Triangular Baſe.

FIND a mean Proportional between the Baſe and half the Perpendicular, or between the Perpendicular and half the Baſe, both meaſured in Inches; this Mean is the Side of a Square equal to the Triangle; then ſet 12 to the Length in Feet, againſt this Side is the Content.

If two Sides of a Triangle be equal, the unequal Side may be the Baſe; if the three Sides be unequal, the longeſt Side is commonly the Baſe; from whence the neareſt Diſtance to the oppoſite Angle is the Perpendicular.

PROP. VIII.

Solids whoſe Baſes have many equal Sides and equal Angles.

THESE Baſes are regular Figures: Having the Length in Feet, and a Side in Inches, get he Perpendicular from the Center to a Side alſo in nches; ſo ſhall the mean Proportional between he Perpendicular and half the Sum of the Sides e the Side of a Square equal to the Baſe; which aving found, meaſure it as ſquare Timber.

Example. A Piece of Timber of eight Sides is o Foot long, the Side 12 Inches, the Perpendiular 14.48 Inches, which you may call 14½; ſet 4½ to 14½, againſt 48, half the Sum of the Sides n the Line of Numbers, is 26.4 on the ſquare ine, or there make a Mark; then ſet 12 to 10, gainſt this Mark is 48 Foot, and a little more han a Quarter.

And thus much of theſe Ways of Timber Meaſure, which being the main Occaſion of the Rule, and not depending on any thing which followeth, I have ſet in the firſt Place.

PROP. IX.

Having the Girt, to find the Side of the Square equal near.

THIS and the two following Propoſitions are wrought on the double Scale, yet I have here adjoined them for their Affinity with Timber Meaſure, and the proportional Numbers given in them are ready cut on the Rule, and give Contents to an Exactneſs ſufficient in any Concerns of Timber.

As 7, to 7.9, ſo the Girt to the Side of the Square equal: Let the Girt be 15 Inches on the double Scale, ſet 7. to 7.9, againſt 15 is 16.9 near: If you ſet 12 to the Length in Feet, this Side ſhall point out a true cylindrical Content.

PROP. X.

Having the Girt, to find the Side of the Square within near.

AS 10 to 9, ſo the Girt to the Side of the Square within near. *Example.* Set 10 to 9, againſt 15 is 13 $\frac{1}{2}$; and ſo much will ſuch a Piece bear ſquare.

By which you may know, before a Piece be hewn, how many whole Boards or Planks, of any Thickneſs, may be had out of it.

From hence alſo you may ſee, that the Girt, tho' leſs than the Side of the Square equal, yet is greater than the Side of the Square within; toward which moſt Timber is hewn before it can ſerve to any ſquare Uſes; which may be one Reaſon of the Continuance of the ſaid common Way; f which Opinion I find alſo Mr. *Henry Philips* to be, in a Treatiſe on this Subject.

PROP. XI.

Having the Diameter, to find the Side of the Square within near.

AS 1 to .707, or to excuſe a Cut there, as 8.5 to 6, (being Points equivalent near) ſo the Diameter to the Side of the Square within near.

Let the Diameter be 19.1, *viz.* the Diameter of o Inches Circumference; ſet 8.5 to 6, againſt 9.1 is 13.5 near.

PROP. XII.

o find how many Inches in Length make a Foot Solid, at any Girt or Side of Square not exceeding 40 *Inches.*

SET the Girt or Side of the Square on the ſquare Line to 1 at the Beginning of the Line Numbers, againſt 41.57 are the Inches which ake a Foot.

So if you ſet 6, for Example, to 1, againſt 41.57 48, and ſo many Inches in Length make a Foot ſix Inches Girt or Side of Square.

And now having done with the Meaſure of ornary Timber, let me advertiſe any Reader that th not ſeen much meaſured, that ſometimes he ll find a great Difference in the Girt of a Tree the Space of a Foot, more or leſs, for the moſt rt where one or more Arms have been cut off; ſuch Caſe it is neceſſary to girt the Tree twice, nay thrice if there be Cauſe, otherwiſe there wil be Loſs to Buyer or Seller.

Alſo they ſay, the Buyer hath Privilege to gir any where between the Middle and the Ground End, if it be for his Advantage.

PROP. XIII.

True Meaſure of a Solid that tapereth ſtrait.

MEaſure the Length in Feet; note alſo the thir Part, which you may find by ſetting 3 on th double Scale to the Length; againſt 1 is the thir Part. If the Solid be round, meaſure the Diameters at each End in Inches; ſubtract the leſs Diameter out of the greater; half the Difference ad to the leſs Diameter; the Sum is the Diameter i the Middle of the Piece.

1. Set 13.54 to the Length, againſt the Diameter in the Middle is a fourth Number.

2. Set 13.54 to the third Part of the Lengt againſt half the Difference is a fourth Number both theſe fourth Numbers added together mak the Content.

Example. Let the Length be 18 Foot, the thir Part is *6*; let the greater Diameter be 24, the le 16, the Difference is 8; half the Difference 4 added to the leſs Diameter 16, the Sum 20 is the Diameter in the Middle.

Set 13.54 to 18, againſt 20 is 39.27. 39.27
Set 13.54 to *6*, againſt 4 is .524, which .52
added to 39.27, maketh 39.794, or 39 39.79
Foot and above 3 Quarters.

Note, That 13.54 muſt be ſet to *6* in the ſecon Length, as in the ſecond Caſe of round Timbe Meaſure the common Way.

If the Solid be ſquare, uſe the Sides of th Squares at each End in every Reſpect as the Di

meter

eters, meaſuring them in Inches, &c. but let 12 your firſt Number.

If it be any other regular Figure, uſe the Sides the Squares equal to each Baſe (found as is be- re ſhewn) as the other, taking alſo 12 for your ſt Number.

PROP. XIV.

The Meaſure of a Shell or Flitch of Timber.

F a Piece be taken out of the Middle of a round Piece of Timber from End to End, there will be t two Pieces, which they call Shells or Flitches.

To find a near Content of theſe after the com- on Way with little Trouble, meaſure the Length Feet, the round Part, and the Thickneſs in the iddle (taken with a Pair of Calepers) in Inches.

Theſe two, with a third Part of the Thickneſs, d together, a fourth Part whereof account your irt, and meaſure as round Timber the common ay.

If on the double Scale you ſet 3 to 4, againſt e Thickneſs is itſelf, with a third Part added it.

Example. Let the Length be 30 Foot, the und Part 25.3 Inches, the Thickneſs 7.2 Inches, t 3 to 4, againſt 7.2 is 9.6, which added to 25.3, aketh 34.9, the fourth Part whereof is 8.7 near.

Or prick the ſaid 9.6 on the flat Part in the iddle from one Side, and keeping the End of ur Line at the other, girt the whole round Part, d to the ſaid Prick; double the Line twice, and eaſure it in Inches for your Girt.

Set 12 to 30, againſt 8.7 is 15 Foot and three uarters.

Note, That this holds not ſo well in Sections cut r from the Middle of the Piece; in others it giveth a Content ſomewhat leſs than the commo Way, which may the better be born with, becau there is more Loſs in theſe than in other Pieces and as they fall ſhort of the middle Pieces in Valu ſo a leſs exact Meaſure may ſerve.

PROP. XV.

Having the Diameter, to find the Area or ſuperf cial Content of the Circle.

SET 1 on the ſquare Line to .7854, or to excu a Cut there, ſet 11 to 9.5, againſt the Diamete is the ſuperficial Content.

Example. Let the Diameter be 1.7 Foot, ſet 1 to 9.5, againſt 1.7 is 2.27 Foot near.

By the Logs. to the Log. of the Diameter twice add this Log. 9.89509, being the Log. of the Decimal .7854, the Sum is the Log. of the ſuperficial Content.

1.7	{ 0.2304 { 0.2304
.7854	9.8950
2.27	0.3559

Note, That if the Diameter exceed not 3.57, in the Middle is but 1; but if it exceeds 3.57, in the Middle is 100.

Here you may have occaſion to make uſe of t Direction given in *Sect.* 5. *Part* 2.

Hence it is as eaſy, having the ſuperficial Co tent, to find the Diameter, or to caſt any Numb into a Circle, ſo may the Gauge Points be put o for they being the Diameters of Circles, who Area's are equal to the Number of Cubick Inch in the Gallon of Wine or Ale reſpectively, if yo ſet the Rule as above, you will ſee the Wine Poi ſtand againſt 231, and the Ale Point againſt 282.

PRO

PROP. XVI.

Cask Gauging.

THE Figures of theſe Veſſels being uncertain,
the Staves of ſome being more circular from
ead to Bung, and ſo more capacious than other,
e late Gaugers diſtinguiſh them into four kinds ;
e Sphæroid, whoſe Staves are moſt arching, and
is contains moſt ; the Conick, whoſe Staves from
ead to Bung are ſtrait, (if any ſuch can be made)
d this contains leaſt ; the Parabola, whoſe Staves
e arching, but nearer to the Sphæroid than to
e Conick , the Conoid, whoſe Staves are arch-
g, but nearer to the Conick than to the Sphæ-
id ; all theſe may have the ſame Dimenſions of
ngth and Diameters, yet differ conſiderably in
e Contents

Mr. *Everard* in his abſolute Piece, *Stereometry*
ide eaſy, printed 1684, giveth a Diagram of all
e Kinds, and Rules for each, both by Arithme-
k and the ſliding Rule, except for the Conick,
re being (as he ſaith) none ſuch made ; yet the
gure thereof is uſeful to the diſtinguiſhing the
er.

Mr. *Wingate* took no Notice of theſe ſeveral
nds ; his general Way applied to this Rule is
us :

Meaſure the Length of the Veſſel within, the
ameter at the Bung, and the Diameter at the
ad in Inches and Tenths, ſubtract the Diameter
the Head out of that of the Bung, the Diffe-
ce multiply by 7, and divide the Product by 10
the Rule eaſily thus ; on the double Scale ſet
to the Difference, againſt 7 is the Quotient,
ich is 7 Tenths of the Difference.

Theſe added to the leſs Diameter, the Sum an æquated Diameter.

Then ſet the Gauge Point, whether of Wine Ale, to the Length, againſt the æquated Diame is the Content in Gallons.

Mr. *Everard* agreeth with Mr. *Wingate* up 7 Tenths near for ten Inches Difference of Diam ters, and accounts them to the Sphæroid; in oth Differences of Diameters they differ more.

His Numbers for 10 Inches Difference are; the Sphæroid 7.01, for the Parabola 6.39, for t Conoid 5.62.

In other Differences the Rule differs ſomethi from his Table, to which I refer you.

To find the Content of a Cask in all theſe Kind let the Length be 34.5, the Diameter at the Bu 29.4, that at the Head 25.3, this deducted of that at the Bung, the Remainder or Differen is 4.1.

Set 10 to 4.1, againſt 7 is 2.87 for the Sphæroi againſt 6.39 or 6.4 is 2.62 for the Parabola; agai 5.62 is 2.3 for the Conoid.

Theſe added ſeverally to 25.3, the Sum is 28. for the Sphæroid, 27.92 for the Parabola, 27.6 the Conoid, for æquated Diameters.

Example. Set the Ale Point to 34.5, agai 28.17 is 76.25 Gallons for the firſt; againſt 27. is 74.9 Gallons for the ſecond; againſt 27.6 is 7 Gallons for the third; ſuch is the Difference (up account of their Shape) by theſe Numbers.

By the Logs. to this general Log. 7.53148 for Wine, to this 7.44484 for Ale, add the Log. of the Length, and the Log. of the æquated Diameter twice, the Sum is the Log. of the Content, as you ſee here for Wine.

Gen. Log.	7.53
34.5	1.537
28.17	1.449 1.449
93.08	1.968

The Fractions are thus reduced to Pints; on the double Scale ſet 10 to 8, againſt the Decimal are the Pints anſwering.

The Sphæroid may be known by the round ſwelling of the Staves from one Head to another. If you lay a ſtrait Rule on the Hoops of a Cask from the Head toward the Bung, and it toucheth, or very near, the Hoop next the Head, and that next the Bung, you may account it a Conoid; if he Rule librate upon the middle Hoops, like the Beam of a Ballance, and yet the Staves not much ſwelling, account it a Parabola.

Beſides the Shape of theſe Veſſels, I have obſerved two Things, not noted by any to my Knowledge, which may render the gauging them uncertain; one is, the joining Staves of unequal Thickneſs, not taking Care to ſmooth them within, which may cauſe an Error of ſome Tenths in taking the Bung Diameter.

The other this; the Head Diameter may be aken too great, tho' taken without, by reaſon of he paring away, and ſmoothing the inward Side f the Cask at each End, in order to the putting n the Heads; ſo that in reaſon it ſhould exceed he Diameter pointed out by the Staves, which is he true Diameter: Both theſe I have ſeen in aſks that have been cut aſunder.

PROP. XVII.

Gauging and Inching of Tuns.

THESE are of ſeveral Figures, but moſt are ſquare or round.

The ſquare are either equal ſided or unequal, oth right-angled, and may be conſidered as the me.

The round are either cylindrical, *viz.* having the Diameters at Top and Bottom equal (if any ſuch can be hoop'd) or conical, whoſe Diameters at the Top and Bottom are unequal.

Alſo the Content may be required, either total, or only of ſome Liquor contained in them.

The Content is ordinarily found firſt in Ale Gallons, which are reduced to Beer Barrels, by dividing the Number of Gallons by 36, or to Ale Barrels, by dividing the ſame by 32; alſo a Barrel containeth 4 Firkins; ſo 9 Gallons of Beer, 8 Gallons of Ale make a Firkin; the Dimenſions, *viz.* Lengths, Breadths, Depths, and Diameters, are taken in Inches.

SECT. I.

Square Tuns.

ON the double Scale ſet 282 (cut on theſe Rules) to either Length or Breadth, againſt the other is the Content in Gallons at 1 Inch deep, which being reduced to Firkins and Barrels, as it will bear, by a continual Addition, as we add Pounds, Shillings, and Pence, a Table may be made to any Number of Inches deep.

Or if you ſet 1 on the ſaid Scale to any Depth in Inches of this Content, againſt the other is the total Content; or multiply them by the Pen.

Example. The Length is 84 Inches, the Breadth 62 Inches, ſet 282 to 62, againſt 84 in the firſt Length is 18.47 Gallons, or 2 Firkins, 2 Gallons, and near an half of Ale, which you may add, as in the Margin.

In.	*B.*	*F.*	*G.*	*Pts*
1	0	2	2	47
2	1	0	4	94
3	1	2	7	41
4	2	1	1	88
5	2	3	4	3
6	3	1	6	82
7	4	0	1	2

Le

Let the Depth be 26 Inches, ſet 1 to 18.47, againſt 26 is 480.22, which is near a Quarter.

By the Logs. To this general Log. 7.54975 add the Logs. of the Length and Breadth, the Sum is the Log. of the Content at 1 Inch deep; and if to this you add the Log of the Depth, the Sum is the Log. of the whole Content; or if to the ſaid general Log. you add the Logs. of the Length, Breadth, and Depth, the Sum is the Log. of the whole Content, without Notice of the Content at 1 Inch deep.

Gen. Log.	7.54975
84.	1.92428
62.	1.79239
18.47	1.26642
26.	1.41497
480.2	2.68139

Becauſe it is likely there will be Tenths of an Inch wet, on the double Scale ſet 10 to the Content in Gallons at 1 Inch deep, againſt every Tenth is his own Share, or Part of the ſaid Gallons. Let he Tenths be 6, ſet 10 to 18,47, againſt 6 is 11 Gallons, belonging to 6 Tenths. They put no Pints into the Table.

SECT. II.

Cylindrical Tuns.

Having the Diameter of a cylindrical Tun in Inches, to find the Content in Ale Gallons at 1 Inch deep.

F the Diameter exceed not 40 Inches, ſet the Ale Point to 1 in the Middle, againſt the Diameter is the Content, 1 in the Middle being one allon.

If the Diameter be above 40 Inches, ſet the ſaid oint to 10 at the End, againſt the Diameter is the ontent, 1 in the Middle being 10 Gallons.

Which Contents, being for 1 Inch deep, may be firſt reduced, and then added continually for a Table; or before it be reduced, multiplied by the Depth for a total Content.

Or ſet the Ale Point to the Depth, againſt the Diameter is the total Content.

Example. Let the Diameter be 58 Inches, ſet the Ale Point to 10 at the End, againſt 58 is 9.37 Gallons, the Content at 1 Inch deep; let the Depth be 36 Inches, ſet the ſaid Point to 36, againſt 58 is 337.3 Gallons; or multiply the Depth and Content at 1 Inch deep by the Rule or Pen.

By the Logs. To the gen. Log. for Ale, add the Log. of the Diameter twice, the Sum is the Log. of the Content at 1 Inch deep.

And if you uſe the Log. of the Depth, as in the former Section, you will have the whole Content.

Gen. Log.	7.44484
58.	1.76343 1.76343
9.37	0.97170
36.	1.55630
337.3	2.52800

SECT. III.

Conical Tuns.

1. TO find the whole Content; proceed as in the Meaſure of a Solid that tapereth ſtrait, *Prop.* 13, only meaſure the Depth alſo in Inches, and inſtead of the Point at 13.54, uſe the Ale Point, as alſo the general Log. uſed in the Section next above, which belongeth to it.

2. But in order to inching them; in ſmall ones and Keelers, they uſe only the Diameter in the Middle, and account them as Cylinders; but in the larger they take one in the Middle of every ten Inches, (beginning at the Bottom) as alſo in the Middle of the remaining Inches, except they be few, for then they account them to the laſt ten,

and take the Diameter in the Middle; theſe Tens alſo they account as Cylinders.

3. Having found the Content anſwering the Diameter next the Bottom, as is ſhewn *Sect.* 2. put it into Firkins and Barrels, as it will bear, and by a continual Addition (as in *Sect.* 1.) make up the ſaid ten Inches.

Then add the Content anſwering the next Dia- neter ſo reduced, Inch by Inch to the laſt Sum, nd ſo proceed till you have finiſhed.

4. In a regular Tun; having the Diameters at Top and Bottom, and the perpendicular Depth, ou may find any intermediate Diameter thus.

Divide the Difference of Diameters by the Depth, he Quotient multiply by any Diſtance from the reater Diameter, and ſubtract the Product from he ſaid Diameter, the Remainder is the Diameter t that Diſtance; or multiply the Quotient by any iſtance from the leſs Diameter, and add the Pro- uct to the ſaid Diameter, the Sum is the Diame- r at that Diſtance.

5. If the Tun be not exactly round, meaſure vo Diameters where you obſerve the Inequality; d them together, and take the half; let the ſaid vo Diameters be the longeſt and ſhorteſt, which ill croſs one another near at right Angles.

6. Becauſe moſt large Tuns are fixed, and that ipping, for the better Deſcent of the Liquor, the uare ones for the moſt Part corner-wiſe, and the owns or Bottoms of the round ones commonly even and irregular, I adviſe you to fill up the d Crowns or Bottoms, as alſo the Crowns of ppers, by Meaſure, till they be wholly covered. Which may be done by a Veſſel of known Quan- y, or you may gauge one or a Pail, or by a true llon, (for making which Directions are after en, which may alſo be otherwiſe ſerviceable) or large Tuns beſt of all by both Veſſel or Pail and

Gallon, uſing the Veſſel firſt, and when near covered, the Gallon.

7. When the Bottom is covered, aſſign the gauging Place, (where the Water covers a whole Inch, if it may be; if not, make it up by Meaſure) and fix it by a Mark, and note the wet Inches; mark alſo the Ends of the Diameters at the Superficies of the Water, as alſo the Perpendicular, or neareſt Diſtance of the Top of the Staff from the Water, where the Diſtance is leaſt, and the Length of the Staff from the Water in the ſame Place; of all which having taken an exact Account, let out the Water, and from the aforeſaid Marks begin the Meaſure of your ſeveral 10 or 12 Inches, and to the Quantity before meaſured in, add your Content Inch by Inch; the Content will be exact enough if you take a Diameter in the Middle of every 1 Inches.

8. Theſe ſeveral 10 or 12 Inches being underſtood to be of the perpendicular Depth, to avoi an Error, which in ſome Caſes may be conſiderable, on the double Scale ſet the perpendicular Dept to the Length of the Staff, againſt any Number the ſaid Depth is the Number anſwering on th Staff, which is always greater than that of the Dept

9. The proportional Parts of any Content b longing to any Diameter, found as before, are be ſet down, every Part againſt its own Tenth, a Column by themſelves, againſt the Contents the whole Inches, to be uſed for the Parts, whi for the moſt Part happen to be over and above t whole Inches.

10. Reduce not the Decimals of a Gallon (t Addition of them being eaſy, and becauſe th make not their Table to Pints) except in the p portional Gallons.

11. The Diameters, whether long or ſhort, meaſured by ſliding Rules, numbered in Inches

th

they are drawn out, (or in Gallons, which will ſave you ſome Trouble) and are made to ſet together (ſo as to be portable) to a great Length.

What hath been ſaid of theſe Tuns, may be underſtood of Coppers, Coolers, or any other Veſſels uſed for Wort, either round or ſquare.

12. In an oval Tun; find a mean Proportional between the longer and ſhorter Diameter, it is the Diameter of a Circle equal to the Oval.

13. As for a true Gallon; to any Diameter in Inches which you chuſe, find the Content in Inches, (as *Prop.* 15.) by which divide 231 or 282 for Wine or Ale reſpectively, the Quotient found to the hundredth Part of an Inch, is the Depth. *Example*, the Diameter is 6 Inches.

The ſuperficial Content anſwering is 28.27, by which dividing 231, the Quotient is 8.17, the Depth of the Wine Gallon, by which again dividing 282, the Quotient 9.97 is the Depth of the Ale Gallon.

If you would have it ſquare, divide the ſaid two Numbers by the Square of the Side in Inches; let the Side be 5 Inches, by 25 divide 231, the Quotient 9.24 is the Depth of the Wine Gallon; and again, by 25 divide 282, the Quotient 11.28 is the Depth of the Ale Gallon.

PROP. XVIII.

To gauge a Stand.

IT may be accounted a cloſe conical Tun, and meaſured as a Solid that tapereth ſtrait, *Prop.* 13. only (as in the conical Tun) meaſure the Depth alſo in Inches, and inſtead of the Point 13.54 uſe the Gauge Points, and the general Log. belonging to them; as in this Example in Ale.

Let the Depth be 33 Inches, a third Part thereof is 11; let the greater Diameter be 30 Inches, the leſs 24 Inches, the Difference is 6, the half Difference 3, which added to the leſs Diameter, the Sum 27 is the Diameter in the Middle.

1. Set the Ale Point to 33, againſt 27 is 67 Gallons.

2. Set the ſaid Point to 11, againſt 3 is .27, or about a Quarter of a Gallon; ſo the Content of the Stand is 67.27 Gallons.

So little is the Difference between the exact Content, and that found by the Diameter in the Middle.

PROP. XIX.

To enlarge or diminiſh a Circle, Square, or other regular Figure at a Rate given.

THE Proportion (reſpecting the Rule) is, as one Term of the Rate to the Square of the Diameter or Side given, ſo the other Term to the Square of the Diameter or Side required; therefore the Root thereof is the Diameter or Side demanded.

Alſo if you would enlarge, the leſs Term of the Rate is firſt; if you would diminiſh, the greater is firſt.

Example 1. If 1000 Men lodge in a Square whoſe Side is 60 Paces, how many Paces ſhall the Side of a Square be wherein 5000 Men may ſo lodge?

Here the ſecond Number being on the firſt or moveable Rule, it is moſt convenient to ſet 60 on the ſquare Line to 1000 in the Middle of the Line of Numbers, againſt 5000 on the Line of Numbers is about 134, and ſo many Paces muſt the Side be.

Example 2. I would diminiſh a Circle, whoſe
iameter is 10 Foot, at the Rate of 8 to 5; ſet
on the ſquare Line to 8 on the Line of Num-
rs, againſt 5 on the Line of Numbers is 7.9 Foot,
e Diameter required.

PROP. XX.

*aving the Dimenſions of the Parts of a Ship,
which make the Faſhion or Shape, together with
the Burthen thereof, to find the Dimenſions of
the ſaid Parts for a Ship of any other Burthen,
greater or leſs, retaining the Faſhion or Shape of
the given Ship.*

HIS Propoſition I find in Mr. *Norwood*, but
wrought with great Trouble. Since the In-
ntion of the Logarithms by the Lord *Napier*,
hoſe Name will never be forgotten) it is per-
rmed with great Eaſe, either by the Line of
umbers, with a Cube Line, being a Line of a
ple Radius adjoined, or moſt exactly by the Logs.
want of the aforeſaid Cube Line, take this way
Compaſſes on the Line of Numbers.

Divide the Space between the Burthen given
d that required into three equal Parts, with this
tent, ſet one Foot of the Compaſſes on each of
given Dimenſions, *viz.* the Length of the Keel,
ngth of the Midſhip Beam, and Depth of the
ld, *&c.*

And if the Burthen required be greater than that
en, turn the other Foot forward to a greater
mber; if leſs, turn it backward to a leſs Num-
, and they will be the reſpective Dimenſions re-
ired in Feet and Tenths.

EXAMPLE.

EXAMPLE.

The Burthen given	100 Tun.	Required 280 Tu
	F.	*F.*
Length of the Keel	50.5	71.2
Len. of the Midſhip-beam	21.	29.6
Depth of the Hold	9.	12.7
Raking forw. of the Stem	13.5	19.
Raking backw. of the Stern	4.	5.6

By the Logs. ſubtract the Log. of the leſs Burthen from the Log. of the greater, the Differenc divide by 3; the Quotient or third Part add to o ſubtract from the Logs. of the ſeveral Dimenſion of the given Ship, according as the Burthen required is greater or leſs than the given; the Sum or Remainders ſhall be the Logs. of the Dimenſions for the Burthen required.

This, holding true in the Dimenſions of Maſts Yards, Cables, Anchors, &c. muſt needs be o great Uſe, being ſo eaſily wrought, eſpecially t the Shipwright, it freeing him from groſs Error and by it he may be inſtructed to provide and o der his Materials to the beſt Advantage.

To Gauge a Cask which is not full.

A TABLE for gauging of Wine Casks which are not full.

G.	parts	G.	parts	G.	parts	G.	parts	G.	parts
0	000	13	2630	26	4330	39	5913	52	7672
1	295		2703		4400		5976		7758
2	470	14	2775	27	4462	40	6040	53	7829
1	602		2847		4542		6094		7909
	720	15	2918	28	4585	41	6158	54	7990
2	830		2986		4646		6223		8072
3	935	16	3056	29	4706	42	6288	55	8154
	1038		3123		4766		6353		8236
4	1138	17	3189	30	4826	43	6418	56	8319
	1235		3255		4885		6483		8404
5	1329	18	3321	31	4943	44	6548	57	8491
	1420		3387		5000		6613		8580
6	1502	19	3452	32	5057	45	6679	58	8661
	1596		3517		5115		6745		8765
7	1681	20	3582	33	5174	46	6841	59	8862
	1764		3647		5234		6877		8962
8	1846	21	3712	34	5294	47	6944	60	9065
	1928		3777		5354		7012		9170
9	2010	22	3842	35	5415	48	7082	61	9280
	2091		3906		5476		7153		9398
10	2171	23	3960	36	5535	49	7225	62	9530
	2242		4024		5600		7297		9705
11	2328	24	4087	37	5662	50	7370	63	10000
	2405		4154		5724		7444		
12	2481	25	4213	38	5787	51	7519		
	2556		4270		5850		7595		

The

The Uſe of the TABLE.

FIND the Content of the whole Caſk, an the Depth of the Liquor therein, being t wet Part of the Bung Diameter, the Axis of t Veſſel being horizontal or level; as the Diamete at the Bung in Inches to the Depth of the Liquo ſo 10,000, the Radius of the Table, to the pro portional Part; find in the Table the ſaid Parts, o neareſt, and note the Gallons and Parts anſwering then as 63, the Gallons in a Wine Hogſhead, t the Gallons noted, ſo the Content of the who Caſk to the Content of the Liquor in the Caſk.

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THE ART OF Practical Meaſuring, &c.

PART IV.

he Uſe of the double Scale of Numbers in ſome ſuperficial Meaſures and Accounts.

Directions.

IN the Rule of Three direct. If the ſecond Number be greater than the firſt, the fourth ſhall be greater than the third; and on the contrary.

But in the inverſe Rule, if the ond be greater than the firſt, the fourth ſhall be s than the third; and on the contrary.

2. If ſetting the firſt to the ſecond, the third reacheth beyond the Line, either remove the firſt to the ſecond in the other Length of the ſecond Line, or take the third Number in the other Length of the firſt Line.

3. The ſecond and third Numbers are never taken both on the ſame Line.

4. Obſerve well what Number goeth with the Queſtion; for in the direct Rule, that of the other two which agreeth with it in Name or Reſpect, is the firſt, which you may ſet to either of the other.

As, if the Queſtion be: If 32 Bricks pave on ſquare Yard, how many Bricks will pave 1 Yards? Here 12 is the third Number, and 1 th firſt, (both being of a Name) which ſet 12 or 3 againſt the other is 384 Bricks; but for the moſ Part the firſt Numbers are given, as you will fin after.

5. It is not hard to know the Value of the fourt Number, for every Number on the Line increaſin or decreaſing in a tenfold Proportion, the Natur of the Queſtion or the Thing meaſured will di cover it; as in the Example above, the fourt Number may be 384, or 38.4, or 3.84; but it evident that it cannot be either of the two latte much leſs 3840; ſo is it in any other.

PROP. I.

Multiplication, 1 *being the firſt Number.*

Sect. 1. *The Square.*

Multiply the Side by itſelf; let the Side be Foot, ſet 1 to 14 (beſt in the firſt Length) agai 14 on the firſt is 196 Foot. This is alſo found the ſquare Line.

Sect. 2. *The long Square.*

Qu. 1. Multiply the longer Side by the ſhorter. A Wall is $30\frac{1}{2}$ Foot long, and 16 Foot high; ſet 1 to 16, againſt $30\frac{1}{2}$ is 488 Foot.

Qu. 2. A Length is 42 Foot, the Breadth $\frac{3}{4}$ Foot; ſet 1 to 42, againſt the Decimal .75 is $31\frac{1}{2}$ Foot.

Qu. 3. How many Men are in a Body, where they ſtand 18 in Front, and 8 deep? Set 1 to 8, againſt 18 is 144.

Sect. 3. *The Triangle.*

Multiply the Baſe and Perpendicular, the one whole by half the other, which you will. In the Pike End of an Houſe, the over-way is 18 Foot, the Diſtance from the Pike to the over-way (being the Perpendicular) 16; ſet 1 to 8, againſt 18 (as before) is 144 Foot; or ſet 2 to either Perpendicular or Baſe, againſt the other is the Content.

Sect. 4. *The Trapezium.*

It is an irregular four-ſided Figure. An irregular Plot (as of Land) before the Content can be found, is divided into theſe Trapezia and Triangles.

To find the Content, draw a Line from one Corner to his oppoſite one thro' the Trapezium, ſo as (if it may be) the two Perpendiculars falling from the other two Angles upon this diagonal Line (as they call it) may fall within the Trapezium; yet if one falls without, the Rule holds, but then the ſaid diagonal Line muſt be produced far enough.

So have you two Triangles having one common Baſe; multiply this Baſe by half the Sum of the Perpendiculars, the Product is the Content of the Trapezium; or ſet 2 to the Baſe, againſt the Sum of the Perpendiculars is the Content.

There is a Trapezium much uſed by the beſt Surveyors of Land, who, when they meaſure againſt a crooked Limit, be it Hedge, Ditch, or River) carry their Chain ſtrait from Mark to Mark, and taking Perpendiculars from the Chain to the Bents. Nooks, or Windings of the Limit, deſcribe Trapezia's in their Plot, have each two parallel Sides, and two right Angles.

The Content is found by multiplying half the Sum of the parallel Sides (being the Perpendiculars) by the neareſt Diſtance between them, being the intercepted Part of the ſaid Chain Line; the Product is the Content.

So is meaſured any Trapezium which hath two parallel Sides, tho' the Angles be not right; but then one Side muſt be continued, if need be, for this Line of neareſt Diſtance muſt be perpendicular to the parallel Sides.

Thus may the Rhombus and Rhomboid be meaſured, and infinite others, neither æquilateral nor æquiangular.

Sect. 5. *Any regular Figure.*

Whoſe Sides being equal, the Angles are alſ equal; multiply half the Sum of the Sides by th Perpendicular let fall from the Center to one o the Sides.

Example. A Table hath 6 Sides, each Side 2 Foot, the Perpendicular 1,73 Foot; ſet 1 to 1.73 againſt 6 is 10,4 Foot.

Sect. 6

Sect. 6. *The Circle, and his Parts.*

1. The ſuperficial Content hereof is beſt found by *Prop.* 15. *Part* 3. It is alſo found by multiplying half the Circumference by the Semi-diameter.

2. For the Semi-circle, multiply half the Arch Line by the Semi-diameter.

3. The Sector, which is any Part contained between two Semi-diameters and the Arch Line, is alſo meaſured the ſame Way.

4. If a ſtrait Line be drawn thro' a Circle, not thro' the Center, it divides the Circle into two Segments; the Meaſure of the leſs is thus: Meaſure the Sector, whereof the Segment is a Part, then ſubtract the Content of the triangular Part, the Remainder is the Content of the Segment; but in the greater Segment the Content of the included Triangle muſt be added.

5. Having the Chord (*viz.* the ſtrait Line above-mentioned) of a Segment, and the Part of the Diameter intercepted between the Chord and the Arch, to find the whole Diameter.

As the intercepted Part of the Diameter to half the Chord, ſo the ſaid half Chord to the other Part of the Diameter; add them, and you have the whole.

6. The Diameter and Circumference are as 7 and 22; ſet 7 to 22, againſt any Diameter is his Circumference; ſet 22 to 7, againſt any Circumference is his Diameter; or having ſet 7 to 22, againſt the Circumference on the ſecond is the Diameter on the firſt.

Sect. 7. *To reduce the aforeſaid Figures to Squares.*

Some of them, as the Triangle, long Square, *&c.* are reduced, as is ſhewn in the Meaſure of

Timber of ſuch Baſes; the other, as alſo any irregular Figure, thus: Firſt find the ſuperficial Content, then ſet the ſquare Line to his Squares, as is before taught, againſt the ſuperficial Content on the Line of Numbers is the Side of the Square.

PROP. II.

Diviſion, wherein the Diviſor is the firſt Number.

Qu. 1. IF 32 Bricks pave one ſquare Yard, how many Yards will 500 Bricks pave? Set 32 to 500, or to 1, againſt the other is 15.6 Yards.

Qu. 2. If 25 Trees coſt 21 *l.* what doth 1 Tree coſt? Set 25 to 21, being neareſt, againſt 1 is 0.84 *l.* whereof the 8 is 16 Shillings, and the 4 is *9 d.* $\frac{1}{2}$, as you will after ſee.

Qu. 3. The Content of a Rectangle or long Square being divided by one Side, (whether the longer or ſhorter) the Quotient is the other. Suppoſe 144 Men placed 24 in Front, how many deep do they ſtand? Set 24 to 144, againſt 1 is 6.

PROP. III.

Sect. 1. *The Rule of Three direct.*

WHEN the Length is meaſured in Feet, the Breadth in Inches, and yet the Content required in Feet, 12 is the firſt Number, marked as 12 on the ſquare Line, chiefly for the Meaſure of Plank, or Board, and Glaſs.

Qu. 1. A Plank is 36 $\frac{1}{2}$ Foot long, 18 Inches broad; ſet 12 to either Length or Breadth, hereto 18, being neareſt, againſt 36 $\frac{1}{2}$ is 54 $\frac{1}{2}$ Foot.

Qu. 2. A Board is 14 Foot long, 26 Inches broad; ſet 12 to 14, againſt 26 is 30 ⅓ Foot.

Qu. 3. A Pane of Glaſs is 2 ¼ Foot long, 7.6 Inches broad; ſet 12 to 7.6, againſt 2 ¼ is 1.42 Foot, which .42 is almoſt an half.

Sect. 2. *Sawyers Meaſure.*

They account 120 to the Hundred. If you would know the Content of a Stock of Plank or Board in ſuch Meaſure; having found the Con-ent of one Plank or Board by the Section aforegoing, ſet 120 (repreſented by 12) to the ſaid Content, or to the Number of Karſes or Cuts, (which are always leſs by one than the Number f whole Boards in the Stock) againſt the other re the Sawyers Hundreds, which will fall in the ſecond Length, except there be not 100 Foot in he Stock. The Tenths are each of them 12 Foot.

Example. Admit there were 22 Boards in the tock mentioned *Qu.* 2. of the foregoing Section, he Content of one Board is 30 ⅓; ſet 12 to 21, he Number of Karſes, againſt 31 ⅓ is 5.31 near. Which is 5 hundred and 37 Foot; for every Tenth being, as is ſaid, 12 Foot, if you ſet 10 to 2, againſt 31 is 37 and more.

Sect. 3. *Glaſs.*

It is moſt convenient in Glaſs to meaſure the Length as well as Breadth in Inches; yet the Con-nt being required in Feet, 144 (repreſented by .4 is your firſt Number, and the Content, if a hole Foot, or above, in the ſecond Length, as xt before.

Example. A Pane of Glaſs is 31 ½ Inches long, broad, ſet 144 to 8 ½, againſt 31 ½ is 1.86 Foot.

Let there be 7 ſuch Panes, ſet 1 to 1.86, againſt 7 is 13 Foot.

Sect. 4. *By the Yard.*

In the following Queſtions both Length and Breadth are to be meaſured in Foot Meaſure. I the Content be required in Yards, 9 is the firſ Number, there being 9 ſquare Feet in a ſquare Yard. So they meaſure Painting, Paving, Plaiſtering, Wainſcot, *&c.*

Example. A Length is 24 Foot, the Breadth $10\frac{1}{2}$; ſet 9 to $10\frac{1}{2}$, againſt 24 is 28 Yards.

Sect. 5. *By the Square of* 10 *Foot, as in Tiling, Flooring*, &c.

Here 100 is the firſt Number. A Roof is 4 Foot long, and the Sparr $20\frac{1}{2}$ Foot; ſet 100 t 41, againſt $20\frac{1}{2}$ is 8.4 Squares.

Sect. 6. *By the ſquare Rod, at* $16\frac{1}{2}$ *Foot to the Rod, as in Brick Walls.*

Here $272\frac{1}{4}$ (being the Square of $16\frac{1}{2}$) is the firſ Number; alſo 272 being cut on theſe Rules, ma ſerve without conſiderable Error. A Wall is 11 Foot long, $9\frac{1}{2}$ Foot high; ſet 272 to 110, againſ $9\frac{1}{2}$ is 3.83 Rod in the ſecond Length. If yo would have a Mark at 324, the Square of 18, ſe 18 to 10, againſt 18 on the ſecond Mark the Poin

Sect. 7. *By the Acre.*

In Land Meaſure 160 ſquare Perches, Poles, o Rod (commonly at $16\frac{1}{2}$ Foot, in ſome Places 1 Foot to the Pole) make an Acre, therefore 16 (repreſented by 16) is the firſt Number; the Par re 40 Pole to the Rood or Quarter, 80 to 2 Roods, 120 to three; the Lengths and Breadths re meaſured in Poles.

Example 1. A Length is 35 Perches, the Breadth 9; ſet 160 to 19, againſt 35 is 4.15 Acres.

Example 2. A triangular Piece of Land hath he Baſe 24 Poles, the Perpendicular $16\frac{1}{2}$; ſet 60 to $16\frac{1}{2}$, againſt 12 (half the Baſe) is 1.24 cre near; or ſet 320 to the Baſe 24, againſt the erpendicular $16\frac{1}{2}$ is 1.24, as before.

PROP. IV.

The inverſe Rule.

HERE the Number that goeth with the Queſtion is the firſt Number, which you may ſet either of the other.

Example. It ſeems $272\frac{1}{4}$ Foot make a Rod of rick Wall at only $1\frac{1}{2}$ Brick Thickneſs; if it be hicker, fewer Feet anſwer a Rod; if thinner, hen more, at an inverſe Proportion.

If it be demanded how many Feet anſwer a od (for Example) at two Bricks Thickneſs,

Set 2 (which goeth with the Queſtion) to $1\frac{1}{2}$, gainſt $272\frac{1}{4}$ is about 204, *viz.* 204.18 Foot; and for any other Thickneſs, which may be marked r firſt Numbers thus: ſet $1\frac{1}{2}$ to any Thickneſs, gainſt $272\frac{1}{4}$ on the ſecond mark the Points.

PROP. V.

Fractions.

BY Fractions I mean Decimals; a general Rule for them is: Set 10 or 100 to the Number of arts (that make the whole) in the Queſtion, againſt every Decimal is its own Share or Portio of the ſaid Parts.

Sect. 1. *Of a Pound Sterling.*

The firſt Figure after the Prick in any Decima of a Pound, is ſo many two Shillings, double i therefore, and you have the Shillings anſwering 5 in the next Place is one Shilling; theſe bein accounted, ſet 100 to 24, againſt the remainin Decimal are the Pence; if there be not 5 in th ſecond Place, having ſet as above, againſt th other is the Pence, the Farthings being very eaſil eſtimated on the Rule.

Example. Let 688 be the Decimal, the 6 i 12 *s.* and 5 in the next 8 is 1 *s.* ſet 100 to 2 againſt 38, the remaining Decimal, is 9 *d.* in a 13 *s* 9 *d.*

Or without the Rule thus: Having taken ou the Shillings, as above, if the remaining Decim exceed not 30, account them Farthings, abatin one; if it exceed 30, take 25 out of it, which 6 *d.* and the Remainder account Farthings, aba ing one.

Note, That the Decimal is ſuppoſed of thr Places at leaſt; if it be but of two, ſuppoſe a Cy pher for the third; and if there be more, yo may neglect them, with the Caution in the lik Caſe before given.

So taking .25 out of .38, (in the Example above the remaining 13 account 12 Farthings, or 3 *d.* the whole is 9 *d.*

Sect. 2. *Of a Rod.*

At 30 *s.* the Rod, what are 28 Cents? Set 10 to 30, againſt 28 is 8.4, *viz.* 8 *s.* and 4 Tenths; ſe 10 to 12, againſt 4 is 4 *d.* ¾; in all 8 *s.* 4 *d.* ¾.

Sect.

Sect. 3. *Of an Acre.*

How many Perches are 15 Cents of an Acre? t 100 to 160, againſt 15 are 24 Perches. More xamples are needleſs.

Sect. 4. *Vulgar Fractions into known Parts.*

Set the Denominator, being the lower, to his umerator, againſt the Number of known Parts the whole is the Number of Parts required; ſet the Denominator to the Number of Parts, gainſt any Numerator is his Portion of the ſaid arts.

How many Farthings are $\frac{5}{7}$ of a Shilling? Set 7 48, againſt 5 is 34, or 8 *d.* 2 Farthings.

Sect. 5. *Vulgar Fractions into Decimals.*

Set the Denominator to his Numerator, againſt 00 is the Decimal required. What Decimal of a oot is $\frac{7}{12}$ or 7 Inches? Set 12 to 7, againſt 100 .58 near enough, or .583.

The aforegoing Examples being well underood, it will not be difficult to apply the double cale to any other Subject.

THE

THE ART OF Practical Meaſuring, &c

PART V.

SECTION I.

The diagonal Scale. (Fig. III.)

IT conſiſteth of 21 equidiſtant paralle Lines thorough the Length of the Scale and of tranſverſe Parallels at a Quarte of an Inch Diſtance one from the other with 5 Diagonals thorough the uppermoſt Integer In the Parallel of 10 there is a Cypher at every Tranſverſe. This Row of Cyphers divides th whole Scale into two Scales, having one diagona Integer over both.

It is fitted chiefly to *Gunter*'s Chain, which is accounted the beſt for ſurveying of Land, and is of 16 Perches in the Inch.

If the Place of Tens in the Links be 0, 2, 4, 6, or 8, (the laſt four whereof are ſet againſt their reſpective Diagonals) uſe the firſt or left Hand Scale; but if the Place of Tens be 1, 3, 5, 7, or 9, uſe the ſecond or right Hand Scale; for that Diagonal which is 20 (for Example) in the firſt, is 30 in the ſecond; that which is 40 in the firſt, is 50 in the ſecond, *&c.*

Example 1. Let 10 Chains and 46 Links be required from the Scale, ſet one Foot of the Compaſſes on the long Parallel or Link Line, repreſenting 6 in the 10th Chain Line or Tranſverſe, and firſt Scale, and extend the other to the Diagonal 40 in the ſaid Link Line of 6.

2. But to take off 10 Chains and 56 Links, ſet one Foot on the Link Line of 6 in the ſaid tenth Chain Line, and ſecond Scale, and extend the other to the ſame Diagonal in the ſaid Link Line of 6, ſo have you the Lines required.

3. So having a Line on your Plot, to know how many Chains and Links it is, take it with your Compaſſes, and carry it parallel to the Link Lines, one Foot in one of the Chain Lines, and the other thorough the diagonal Integer, till it falls on one of the Diagonals, and according as it falls in the firſt or ſecond Scale, ſo account the Tens of your Links.

The Scale may be made to 20 in the Inch, (as it is commonly to 12, and ſet on the other Side) which muſt needs exceed any plain Scale of that Dimenſion for Exactneſs.

SECT. II.

Gunter's Chain.

IT is 4 Perches long, at 16 $\frac{1}{2}$ Foot to the Perch, theſe make 792 Inches; it hath 100 Links, ſo each Link is 7.92 Inches.

Theſe Chains are diſtinguiſhed with Pieces of Braſs at every tenth Link, which Pieces contain ſo many Corners or Points as they are Tens of Links diſtant from either End of the Chain, thus: That Braſs at 10 Links has one Tippet or Point, that at 20 has two, that at 30, three; that at 40, four; and the ſame again from the other End; but at 50, the Middle, is a large round Piece of Braſs; and at 25, from each End, two Curtain Rings together. By the Help of theſe Diſtinctions (which are plainer, and far more viſible than the old Way of Rings alone) you will ſpeedily find the Number of Links.

Altho' the Chain be divided into 4 Perches by the two double Rings and the large Braſs Circle in the Middle, ſo that it may be applied to the Meaſure of any Length by the Pole, yet in meaſuring Lengths in ſurveying we take Notice only of Chains and Links, not concerning ourſelves with Perches till we caſt up the Content.

To multiply a Length and Breadth meaſured with this Chain, reduce them into Links, which is no more Trouble than to ſet the Links at the right Hand of the Chains; or if there be no Links, to put two Cyphers there; ſo 4 Chains and 32 Links are 432 Links, and 7 Chains are 700 Links, 10 Chains and 6 Links are 1006 Links.

Having multiplied a Length by a Breadth, the ſixth Figure of the Product to the left Hand (if there be ſo many) is Acres compleat, the ſeventh Tens of Acres.

Example 1. If you multiply 5 Chains and 82 Links by 3 Chains and 21 Links, the Product is 1.86822, whereof the 1 is one Acre.

Ex. 1.
582
321
582
1164
1.746
1.86822

Alſo 13 Chains and 42 Links by 8 Chains and 70 Links, the Product is 11.67540, whereof the 11 is eleven Acres. The Decimals are reduced to Roods and Perches as followeth.

Ex. 2.
13.42
870
93940
10736
11.67540

SECT. III.

To reduce the Decimal Lines of Gunter's *Chain into Poles.*

AN Acre is 160 ſquare Perches, as hath been ſaid, equal to 100,000 ſquare Links of this Chain, which being divided by 160, the Quotient 625 is the ſquare Links of one Perch or Pole; this, as it is the Decimal of an Acre, ought to be expreſſed thus .00625, as alſo all other, *viz.* with 5 Places, except it be even Tens, for a Cypher or Cyphers at the left Hand are of no Value, as hath been ſaid.

The Parts of an Acre are firſt 4 Roods, whereof 4 is your firſt Multiplier, and there being 40 Pole in a Rood, 40 is your ſecond.

Or if you multiply your Decimal by 160, the Figures remaining to the left Hand, after the Decimal be cut off, are Perches immediately.

But where the Content is not exacted to half a Pole, we uſually take this ſhorter Courſe, without prefixing Cyphers.

It is evident, that if the Places of the Decimal be but three, there cannot be two Poles; if they be four, multiply the firſt Figure by 6; if five, multiply the two firſt Places by 6, in both ſet the Product one Place back toward the right Hand, then add together the firſt, or two firſt Places (reſpectively) of the Decimal and Product ſo ſet, adding alſo a Unit for every 6 that ſhall be in the Figures of the next Place, the Sum is the Number of Poles in the Decimal.

See here two Examples, one of 4 Places, the other of five; alſo you ſee how the Products are placed and added.

Ex. 1. *Links* 8172
48
Poles 13

In the firſt there is once 6, in the ſecond twice 6, in the Figures of the next Place, for which 1 in the firſt, and 2 in the ſecond, are added to the other; the two Places to the right Hand are neglected, as never amounting to the ſixth Part of a Pole.

Ex. 2. *Links* 21726
126
Poles 35

The Reaſon of this Operation, Mr. *Wingate* (in whoſe Arithmetick I firſt met with it, uſed alſo by Mr. *Atwell*, as you may ſee in his Book of ſurveying printed *Anno* 1662, which I value beyond moſt Books of that Subject printed ſince) deeming it not very obvious, leaves to the Search of the Curious. Take it here:

1. If 100 (which are ſo many thouſand ſquare Links being an Acre) require 60 to be added to it to make it 160 Poles, being alſo an Acre, what ſhall any other Number of thouſands of ſquare

Links require to be added to it to turn it into Poles ?

2. To multiply a Number by 6, and divide the Product by 10, gives the ſame Quotient which you have by multiplying the ſame Number by 60, and dividing the Product by 100.

3. Setting the Product a Place back to the right Hand, both divides by 10, and ſeats it for Addition.

SECT. IV.

A ready and exact Way by the Rule.

ON the double Scale ſet 100 to 16, againſt the Links are the Poles anſwering; neglect the two laſt Figures, as is ſaid.

If the Decimal be 60,000, or more, take 5 out of the firſt Figure, accounting for it 2 Roods, and find the Poles anſwering the Remainder; let the Decimal be .82511, deducting 5, the Remainder is .32511; ſet 100 to 16, againſt .325 is 52, *viz.* 1 Rood and 12 Pole; ſo the Decimal is 3 Rood 12 Pole.

SECT. V.

Having the three Sides of a right Line Triangle, to find the ſuperficial Content.

ADD the three Sides together, from half the Sum ſubtract each Side ſeverally, ſo you have three Remainders, multiply theſe three and the half Sum continually, that is, the firſt Remainder by the ſecond, the Product by the third, and this Product by the half Sum, the ſquare Root of this laſt Product is the ſuperficial Content.

Moſt eaſily and ſpeedily by the Logs. thus: Add the Logs. of the three Remainders and the half Sum together, half the Sum is the Log. of the ſuperficial Content.

Example. Let the Sides be 1050, 854, 774, the half Sum is 1339, the three Remains 289, 485, 565, the Content is 3 Acres, 1 Rood, and 1 Perch near: This is the moſt certain Way of meaſuring Land, where the Triangles can be meaſured in the Field; which otherwiſe are firſt plotted on Paper, and the Length of the Baſe and Perpendicular of every Triangle meaſured upon the plotting Scale, and from them the Content caſt up, as has been ſhewn before: Not but there are Methods by which you may caſt up the Content of Lands, tho' ever ſo irregular, without plotting, and that to the greateſt Truth imaginable, as being grounded upon a Theory, as true as any Propoſition in *Euclid*, and where the utmoſt Exactneſs poſſible is required, is doubtleſs the beſt Way.

1339	3.12678
289	2.46090
485	2.68574
565	2.75205
a.	11.02547
3.25640	5.51273
150	*a. r. p.*
41	3. 1. 01 *near*

Here follows the Deſcription of the other Side of the Rule, which is applied to find the Lengths and Angles of Hips, Rafters, and Collar-Beams, whether the Roof be ſquare or bevelling, and that at any Pitch.

FIRST then, on the innermoſt Edge of each Leg or Joint of the Rule, is drawn a Line from the Center, numbered 2, 3, 4, 5, 6, and ſo on to 15, whoſe Uſe is to divide a Circle into any Number of equal Parts not exceeding 15.

Next to theſe Lines on each Joint is drawn another Line, which is divided into 30 equal Parts repreſenting ſo many Feet, each Foot being divided into 12 equal Parts to expreſs the Inches.

There is on one of the Legs a Line divided into 40 equal Parts, of the ſame Length with the 30 Scale, which alſo repreſents Feet, each Foot being divided into 6 equal Parts, and therefore each Diviſion is 2 Inches.

On the other Leg, next the 30 Scale, is a Lin of Chords, numbered 10, 20, 30, and ſo on t 180, each of which Parts are divided into 1 more, to expreſs each ſingle Degree, whoſe U is to find the Quantity of any Angle in Degree and Minutes.

Before we proceed to the Uſe of theſe Lines, it won't be improper to explain what is meant by the Terms Lateral and Parallel.

Firſt, By the Word Lateral is meant any Diſtance taken in a ſtrait Line from the Center, either upon the 30 or 40 Scale, or upon the Line of Chords.

For Inſtance, ſuppoſe I would take 20 Feet laterally off of the 30 Scale, fix one Point of the Compaſſes in the Center, and extend the other to 20 on the 30 Scale of either Leg, and the ſaid Extent is called a lateral Extent of 20 Feet. And ſo for any other.

2. By a parallel Extent, we mean any Diſtance taken by extending the Compaſſes from any Number of Feet and Inches on the 30 Scale of either Leg, to the like or unlike Number on the 30 Scale of the other Leg, as thus: The Rule being any how opened, the Compaſſes extended from 25 on one Leg to 25 on the other, is a parallel Extent of 25 Feet; but if it had been extended from 25 on one Leg, to any other Number more or leſs on the other Leg, it's ſtill called a parallel Extent.

3. When one Point of the Compaſſes is fixed in given Point, and the other ſo opened or ſhut as ſt to touch a given Line, that Extent is called e neareſt Diſtance between the ſaid Point and ne.

4. *How to repreſent any Number of Feet and Inches by a right Line.*

xtend the Compaſſes laterally from the Center he 30 or 40 Scale to the Number propoſed, and is the Length of the Line required.

5. But if theſe Scales are too great or ſmall ; for Inſtance, ſuppoſe you wanted a Line of 4 nches to be divided into 30 equal Parts, or, which s the ſame Thing, to have a Line of 4 Inches epreſent 30 Feet ; proceed thus :

Take 4 Inches between your Compaſſes from off he Scale of Inches, and make the ſaid Extent a arallel in 30 and 30, and thus the 30 Scale is ſet o your Deſire.

6. *To ſet the 30 Scales ſquare.*

Take 90 from off the Line of Chords with our Compaſſes, and make the ſaid Extent a Paallel in 15 and 15 of the 30-Scales, and then the aid Scales are ſet ſquare or perpendicular to each ther.

After which ſhort Deſcription and Uſe of *Scaozzi*'s Lines, let us apply them to that Part of uilding, where the Length of Hips, Rafters, and ngles are concerned, whether the Frames be quare or bevelling, &c.

PROP. I.

LET ABCD, in *Fig.* I. repreſent the Frame of a Houſe bevelling at one End and ſquare t the other, the Gable End being ſquare, and the evel hipt.

In which A B is 11 Feet, B C 14 Feet 5 Inches, D 9 Feet 6 Inches, and A D 20 Feet, and havg drawn the Poſition of the Frame according to eſe Lengths, draw the Gable End D E C, whoſe afters D E and C E are each equal to three ourths of the Breadth of the Houſe C D ; biſect r divide the Lines A B and C D into two equal arts in the Points G and F, and draw the Line

FG for the Ridge of the Houſe; and EF, th common Perpendicular, will repreſent the Heig thereof above the Floor.

Next, make GH equal to BG or GA, an thro' the Point H draw the Diagonal AI and B which will ever interſect each other at righ Angles; and then thro' H draw the Lines P and *mn*, the former perpendicular to FG, an the latter parallel to AB.

To find the Length of the Hips.

Make HI equal to EF (5 Feet 4 Inches) th Height of the Rafters perpendicular, and dra the Lines AI and BI for the two Hips, AI in thi Caſe being the longeſt, and BI the ſhorteſt, whic being raiſed to their proper Pitch, will meet th principal Rafters KP and LS perpendicular t the Point H.

To find the Back of the Hips, ſo that it may anſwer both Sides and Ends of the Roof.

Lay the Ruler from G to *n* and from G to *m* and interſect the Diagonals HB and HA in th Points *u* and *w*; then with one Point of the Compaſſes fixed in the Point *u*, with the other tak the neareſt Diſtance to the Hip BI, and lay it o from *u* to M, and drawing the Lines *n*M, MG they will form the Angle *n*MG, equal 137°, fo the Back of the leſſer Hip BI.

Alſo, if with one Point of the Compaſſes fixed in the Point *w*, you take the neareſt Diſtance to the Hip AI, and lay it off from *w* to M, and draw the Lines *m*M, MG, you will have the Angle *m*MG equal 99°, for the Back of the longeſt Hip AI.

And if you make KP, DQ, LS, and CR each equal to the Rafters CE or DE, and joi

the

e Points Q P and S R, they will be the Length the Ridge of the Roof, each being equal to H. Now, if to P K or D Q, and to L S or R, you draw Lines parallel, about 12 or 14 ches diſtant from each other, at which Diſtance ſeems Rafters ſtand, and allowing for their readth, you will have them repreſented to your iew as they lie in Ledgment; alſo if A N and N (equal to the Hip Rafters A P and B S) are rawn, they will appear juſt in the ſame Manner when they lie in Ledgment.

Vith the Rule to find the Length of the Rafters by Inſpection, when true Pitch.

Againſt 9 Feet 6 Inches (the Breadth of the ouſe) upon the 40 Scale is 7 Feet $1\frac{1}{2}$ Inches up- n the 30 Scale, for the Length of the Rafter; lſo againſt 7 Feet $1\frac{1}{2}$ Inches upon the 40 Scale, 5 Feet 4 Inches upon the 30 Scale, for the Length the Rafter's Perpendicular, or Height of the idge above the Floor.

But if the perpendicular Height of the Gable nd ſhould be agreed on, and by that means the oof be above or under true Pitch, then open the ule, ſo that the 30 Scales may ſtand perpendi- lar to each other, and count half the Breadth of e Houſe upon one Leg from the Center, and e Height of the Perpendicular upon the other, iz. both upon the 30 Scales, and the parallel Di- ance between them meaſured laterally upon e 30 Scale, will give the Length of the Rafters.

EXAMPLE.

Suppoſe the Breadth of a Houſe be 35 Feet, nd the perpendicular Height of the Gable End 3, then working as above directed, and you will find the Length of the Rafter to be 28 Fee 10 Inches.

It's doubtleſs of great Uſe to know what Angl the Rafters make at Foot and Head, with th raiſing Piece and the King's Poſt, and of th Angles made at the Foot and Top of the Hips, an therefore I ſhall give one general Rule to meaſur any Angle.

Always take the Length of thoſe Lines whic contain the Angle you would meaſure, and coun them from the Center on the 30 Scales, and ſ open or ſhut the Rule, that the parallel Diſtanc between them, may be equal to the Side oppoſit to the required Angle, taken laterally off from th 30 Scale by a Pair of Compaſſes; and then if th parallel Diſtance between 15 and 15 be meaſure upon the Line of Chords, it will ſhew the Degrees and Minutes of the Angle.

EXAMPLE.

Suppoſe it was required to meaſure the Angl D E F, *viz.* the Angle made by the Rafter D and its perpendicular Height E F.

Firſt, I take the lateral Extent of 4 Feet Inches in my Compaſſes off of the 30 Scale, whic is the Side oppoſite to the Angle ſought, the apply it parallel, by fixing one Point of the Compaſſes in 5 Feet and four Inches on the 30 Sca of either Leg, and ſo open or ſhut the Rule, th the other Point of the Compaſs may reſt upon Feet 1 ½ Inches on the 30 Scale of the other Leg and then taking the Diſtance with your Compaſſ between 15 and 15, and meaſuring it lateral upon the Line of Chords, the Quantity of t Angle ſought will appear to be 41 Degrees Minutes, which ſubtracted from 90 Degrees, a

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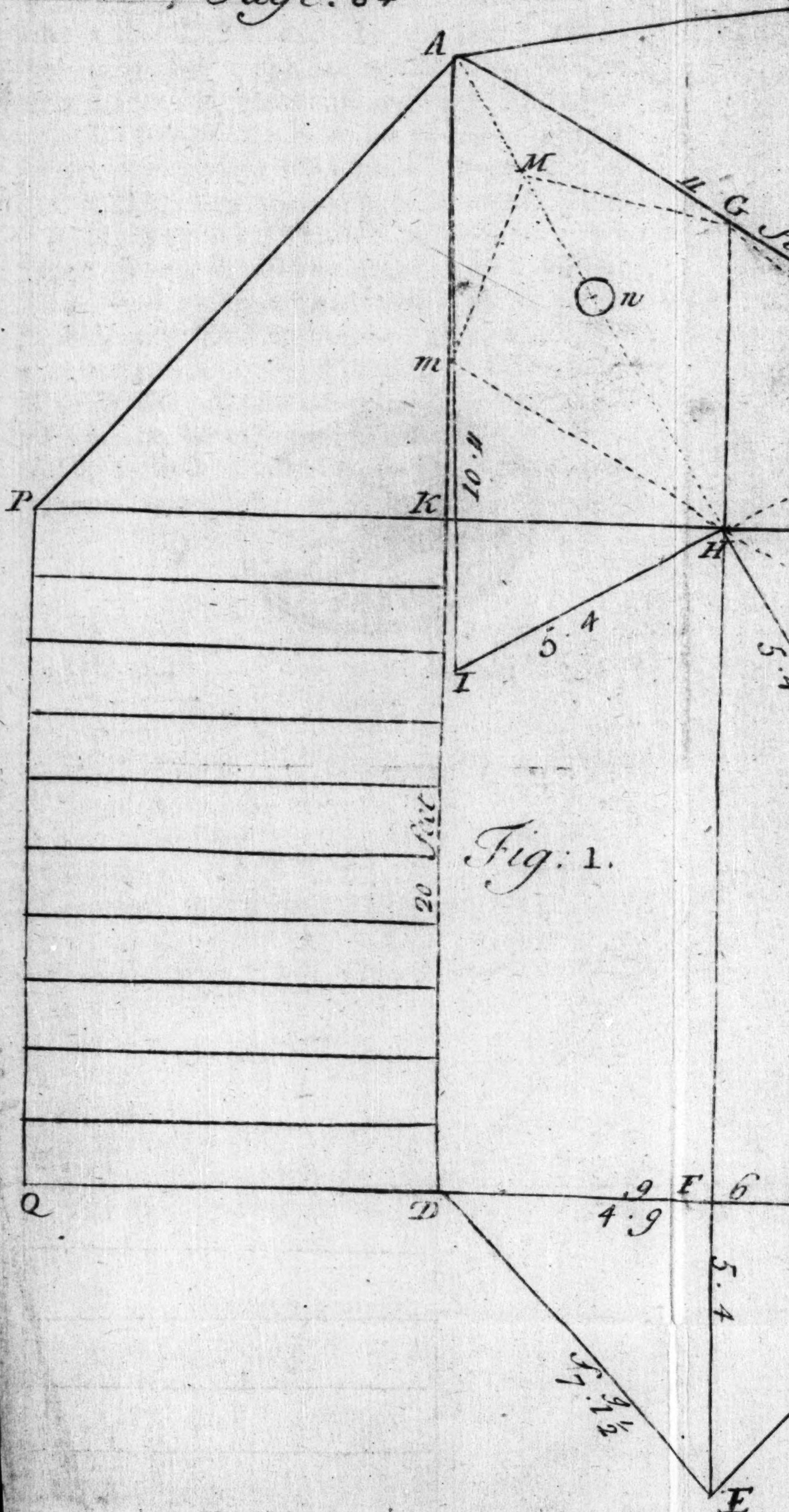

Fig: 1.

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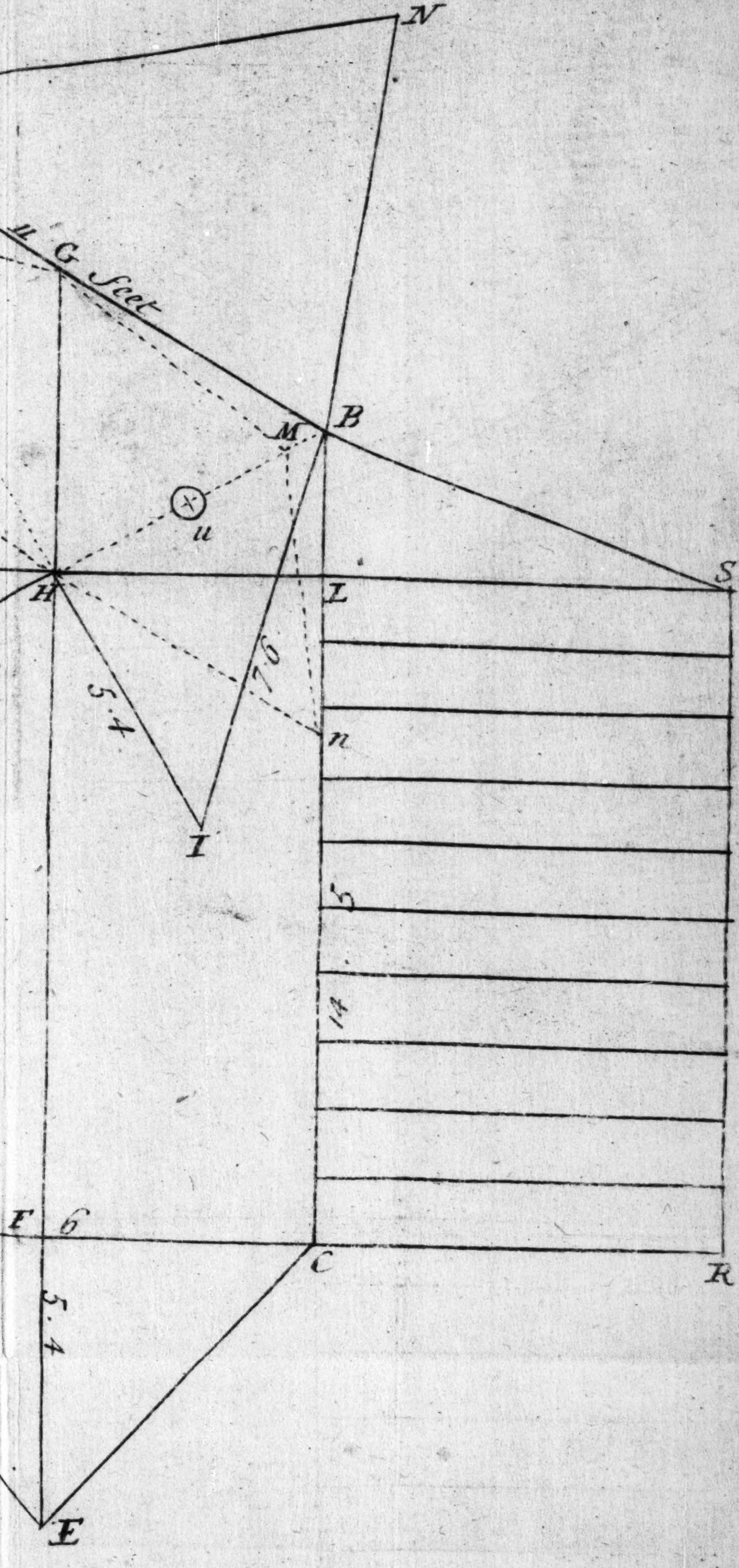
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there will remain 48 Degrees 10 Minutes for the Angle F D E at the Foot of the Rafter.

Or the Angle D E F may be meaſured thus: Lay the innermoſt Edge of either Leg juſt to touch the Line E F, the angular Point E being as near to the Center of the Rule as poſſible for the Braſs Joint; then ſo open or ſhut the other Leg of the Rule, that it may remain parallel to the Line DE, and the parallel Diſtance between the two little Lines drawn upon the inner Edges of the Rule at 15 and 15, meaſured upon the Line of Chords, will give the Degrees and Minutes of the Angle, equal 41 Degrees 5 Minutes, as before.

And after this Manner the Angle at the Back of the Hips may be taken with a Bevel, and ſerve in Practice, as well as if taken by the Rule, and the Degrees and Minutes of the Angle determined.

PROP. II.

To find the Rafters, Hips, and Angles of bevel and taper Frames.

IN theſe Sort of Frames obſerve, that the Middle Breadth is the Guide for the Rafters Length, and that the Perpendicular at both Ends muſt be equal to the middle Rafters Perpendicular, to prevent the Roof's being higher in one Place than another; for tho' a Pair of Rafters at one End of the Houſe will exceed the Length of a Pair at the other, yet, becauſe they and the reſt of the Rafters are proportioned by a common Perpendicular, all Parts of the Roof will be of an equal Height.

In *Fig*. II. Let A B C D repreſent ſuch a Frame, in which, beſides knowing the Lengths of the 4 Sides, it's neceſſary that one of the Angles ſhould be given, in order to lay down the Frame in its true Poſition; for if one of the Angles be not determined, there may with the ſame 4 right Lines be conſtituted an infinite Number of Figures, and all different in Area and Form from one another: Wherefore let the Angle D A B be 70 Degrees.

And then to deſcribe the Frame, having drawn the Line AD equal to 26 Feet, proceed to lay down the Angle DAB and the Side AB, after this Manner; take 70 Degrees, the Quantity of the Angle DAB, laterally off with your Compaſſes from the Line of Chords, and make the ſaid Diſtance a Parallel between 15 and 15; and then fixing one Point of the Compaſſes in 26, (on the 30 Scale of either Leg) which is the Length of the Side AD, extend the other to 11 Feet 9 Inches on the 30 Scale of the other Leg, and with this Extent, one Foot of the Compaſſes being fixed in the Point D, with the other deſcribe the ſmall Arch *r* B *s*; and then taking 11 Feet 9 Inches between your Compaſſes, and fixing one Point in A, with the other deſcribe the Arch *u* B *w*, and where the Arches interſect each other as in B, draw the Line A B, and ſo ſhall the Line A B be laid down according to its true Length and Poſition: And ſince the Points D and B are known, 'twill be eaſy to determine the Point C by the Length of the Lines D C and B C, which are alſo given, D C being 9 Feet 6 Inches, and BC 18 Feet 10 Inches.

Next, divide DC into two equal Parts in the Point Z, and AB in the Point G, and having drawn the Line ZG, which repreſents the Place over which the Ridge of the Houſe muſt ſtand, make G I equal to BG or A G, and Z F equal to D Z or Z C, and thro' the Point I draw the Diagonals A *m* and B *k*, which will always interſect each other at right Angles, tho' the Frame bevel and taper ever ſo much, as may eaſily be proved from the Elements of *Euclid*: Alſo thro' the Point F draw the Diagonals D *n* and C *h*.

Croſs Z G in the Middle at right Angles, as *yx*, which middle Breadth being upon Trial found to meaſure 10 Feet, three Fourths thereof, *viz.* 7 Feet and half will be the Length of the Rafter *b c*: And again, three fourths of the Rafter will produce 5 Feet 7 ½ Inches for the Rafters Perpendicular, o: Height of the Ridge all over the Frame. This may be done by Inſpection by the 30 and 40 Scales on the Rule.

Which being either Way determined, let us proceed to find the Length of the Hips.

Firſt then, becauſe A I and B I repreſent the Diſtance of the King's Poſt, or Point over which the Hips will ſtand, when raiſed to their proper Places from the two Corners of the Frame A and B, and that the I H's are perpendicular to the ſaid Diſtances A I and B I, and both equal to 5 Feet 7 ½ Inches, the perpendicular Height common to all the Rafters.

I ſay, If you draw the Lines A H and B H, it's evident they will repreſent the two Hips at that End of the Houſe expreſſed by the Letters A G B, the greater Hip A H being 11 Feet and 1 Inch, and the leſs B H 9 Feet 6 Inches.

Moreover, if F K be made equal to *y c* 5 Feet 7 ½ Inches, and D K and C K drawn, they will, by being meaſured upon the 30 Scale, ſhew the Length of the Hips at the other End of the Houſe, *viz.* D K the greater Hip 9 Feet 6 Inches, and C K the leſs 7 Feet 9 Inches.

To

To find the Angles for the Back of the Hips.

In order to which, fix one Point of the Compaſſes in I, the Foot of the King's Poſt, and with the other take the neareſt Diſtance to the Hips A H and B H, laying off the former Diſtance from I to M, and the latter from I to N, and drawing the Lines *k* M, M B and A N, N *m*, the Angle *k* M B will be 108 Degrees for the Back of the longeſt Hip A H, and the Angle A N *m* 124 Degrees for the Back of the leſſer Hip B H.

The Quantity of theſe Angles is found by the general Rule; as delivered before for meaſuring of all right Line Angles.

To repreſent the Breadth and Length of each Side of the Roof, raiſe two Perpendicular, *viz.* *a* Q and *d* P from the raiſing Piece A D, and ſo that, if continued both ways, they may paſs thro' the Points I and F; and then lay the Length of the Hip A H from A to P, and of D K from D to Q, and draw the Lines D Q, Q P, and P A, for one Side of the Roof.

Then make *q* T and *p* S perpendicular to the raiſing Piece B C, and ſo that they may likewiſe paſs thro' the Points I and F; then lay the Length of the Hip B H from B to T, and of C K from C to S, and join the Points C, S, T, and B, by drawing the Lines C S, S T, T B, which will form the Shape of the other Side of the Roof; and as Q P and S T are each equal to F I, the Diſtance between the King's Poſts, ſo when they are turned over to meet in their proper Places, will repreſent the Ridge of the Houſe; and if A B V, the greater Hip, and D C E, the leſs, be turned over to their reſpective Places, the Point V will meet the Points P and T, and be perpendicular to the Point I;

and

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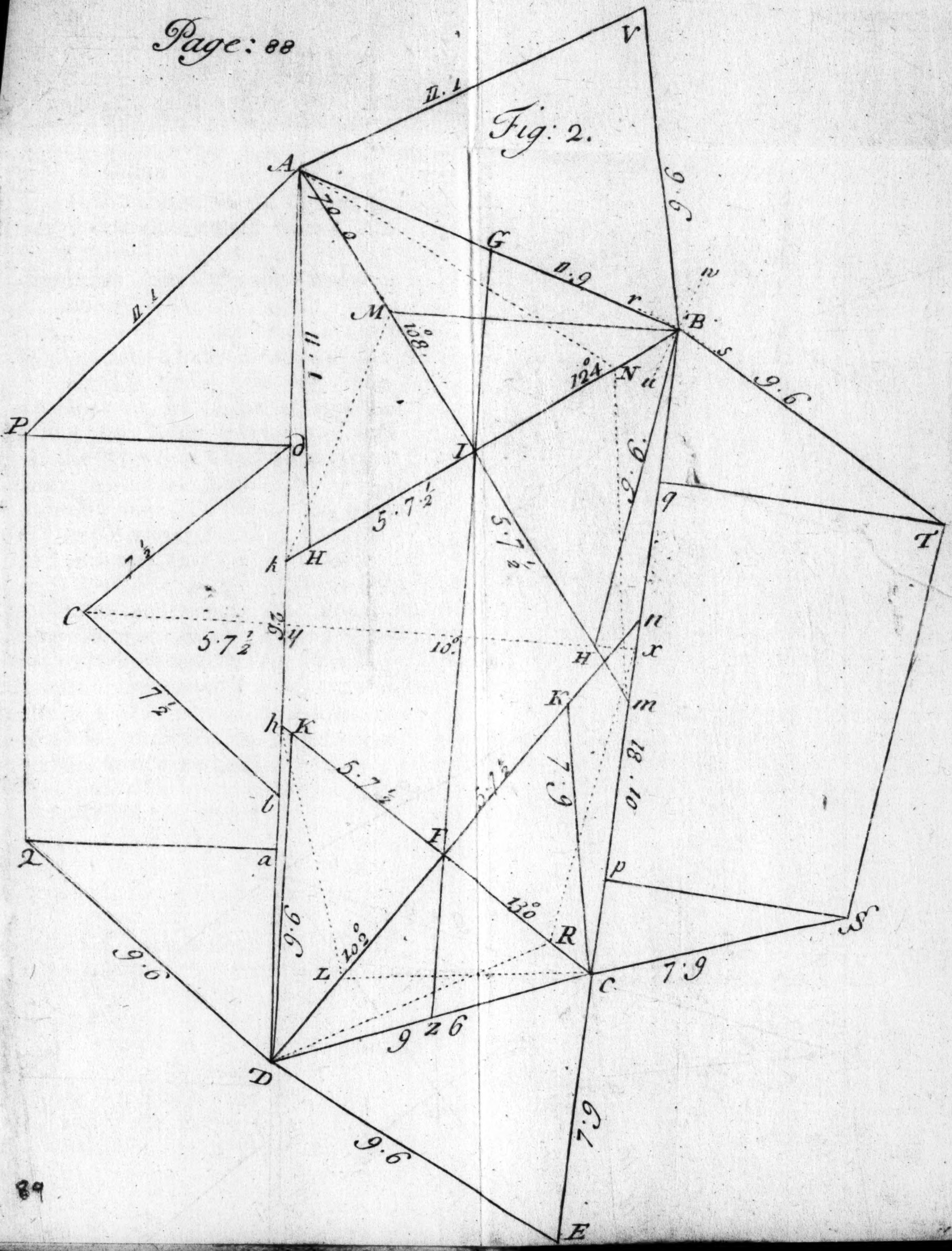
Fig: 2.



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the Point E will meet the Points Q and S,
be perpendicular or right over the Point F.

find the Lengths and Angles of Collar-Beams in any Roof.

After having taken the whole Breadth of the
me between your Compaſſes, ſet one Point
reof in the Length of the Rafter on the 30
le of either Leg, and let the other Point reſt
he Length of the Rafter, counted upon the 30
le of other Leg, which two Legs repreſent the
principal Rafters; and a Rule being laid from
Point of the Compaſſes to the other, repreſents
raiſing Piece; then at any Height you deſign
Collar Beam ſhall be above the raiſing Piece,
ou apply a Ruler parallel to it, the Diſtance
ween the Rafters is the Length of the Collar-
am, (allowing ſpare Wood for the Tenons)
ich Diſtance you muſt meaſure laterally from
Center on the 30 Scale; and as for the Angles
cut the Tenons by, they are the ſame as the
fters make at Foot with the raiſing Piece.

PROP. III.

find the Lengths and Angles of Rafters and Purloins in Bevel Frames.

N *Fig.* III. let A B C D be a Frame bevelling at
one End, and ſquare at the other: Firſt, The
gth of the Lines or Sides of the Frame being
ermined, it will be eaſy (by what has been al-
dy ſaid) to lay it down in its true Poſition;
ich being done, draw A E parallel to B G, and
duce C D to E; biſect or divide B C and A E
two equal Parts at the Points N and P, and
w N P for the middle Line of the Frame, and
draw

draw KH in the Middle between AB and N and MG in the Middle between NP and C and ſo will CK and BM repreſent the Lengt a ſquare Pair of Rafters, each being three Fou of the Breadth of the Frame BC, therefore e to 15 Feet; three Fourths of which is 11 Fe Inches, the Length of the Perpendicular com to all the Rafters.

Next, make FG and IH each equal to and draw the Lines DI and AF, which will preſent the Outſide Lines of the bevel End R ters, and the Lines parallel to them the Inſide, being drawn according to the Breadth or Scantl of the Rafters, which here is 8 Inches.

Thus the Lines AF and DI repreſent the b End Rafters, laid in Legment to fit in the Purlo they lying out of Square according to the Ang GAF and CDI, the one being 11° 20′ leſs t 90°, and the other 11° 20′ more than 90°; wh Angles alſo repreſent the Back of the Rafters the Foot.

And as CK and BM repreſents a Pair of ſqu Rafters at ſome intended Diſtance from A; TS and RQ will repreſent the true Length of Purloins fit for thoſe Places, RQ being the ſh eſt, and TS the longeſt.

And as for meaſuring of them, and finding Angles, which the Workman may think neceſſ in his Practice of Building, they are perform in all Reſpects, as has been before deſcribed in precedent Schemes; I therefore thought it ne leſs to repeat them again, but choſe rather to le the Operations to the Reader's own Applicat and ſo conclude.

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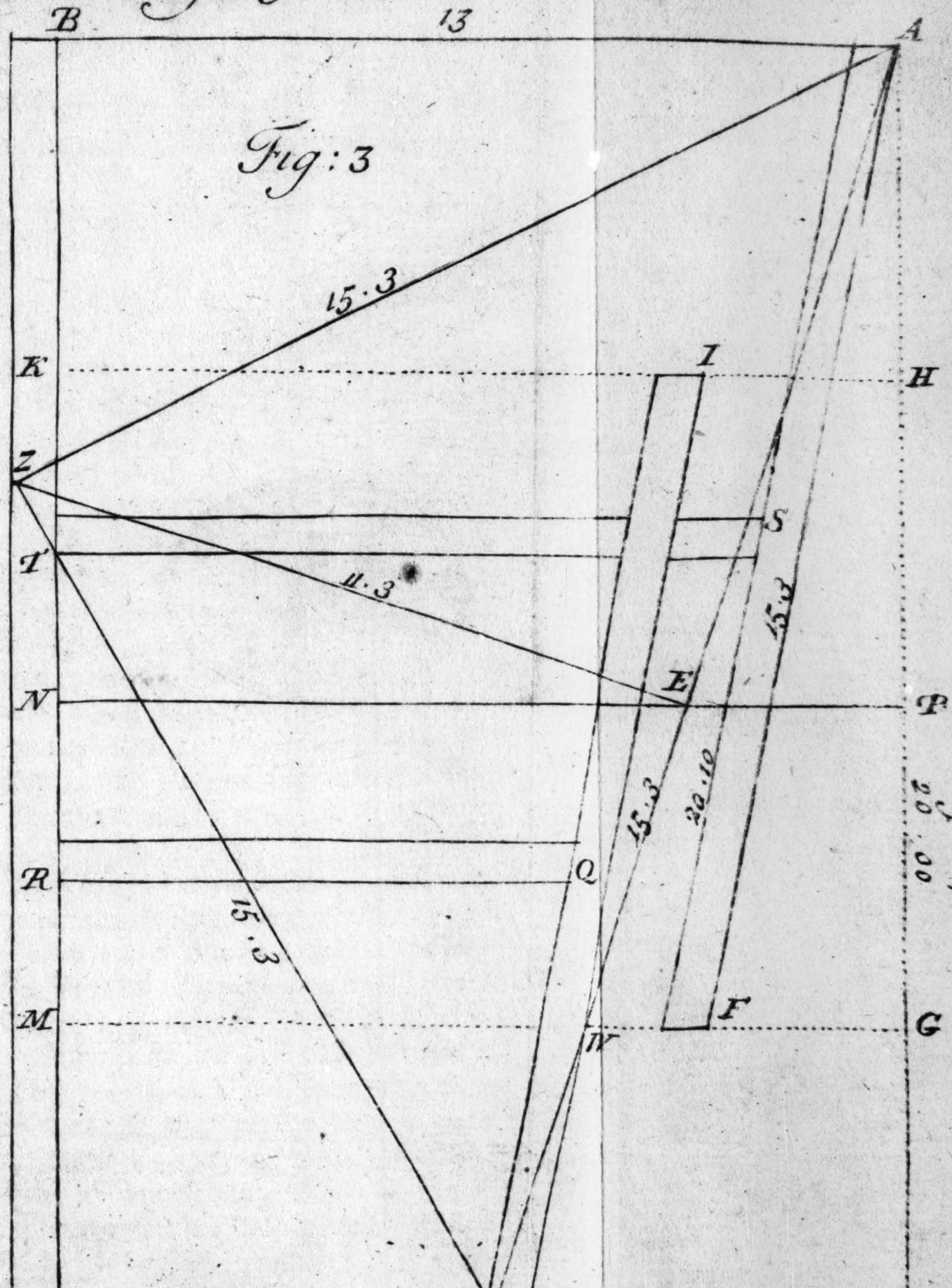

Fig: 3

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ERRATA.

PAGE 7. *line* 7. *for* A *read* As, *p.* 9. *l.* 20. *after* XB *add*, then cut off AC equal to AE, *l.* 25. *for Fig.* 6. *r. Fig.* 5. *p.* 12. *l.* 13. *for* through F, *r.* through Q. *p.* 18. *l.* 7. *for* DG *r.* GB *and for* HG *r.* AG, *p.* 28. *l.* 28. *Marg.* add, *Fig.* 4. *p.* 47. *l.* 4. *for* Sine *r.* Line, *p.* 55. *l.* 1. *add* Sect. IV. *l.* 5. *Marg.* add, *Fig.* 19. *l.* 25. *Marg.* add, *Fig.* 20. *p.* 56. *l.* 21. *Marg.* add, *Fig.* 21.

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DEFINITIONS.

A POINT is that which has no Parts.

2. A Line is Length, without Breadth, and is produced by the flowing of a Point.

3 Points are the Terms of a Line.

4. A right Line, as A B, is that which lies enly betwixt its Terms. *Fig.* 1.

5. Parallels are ſuch right Lines in the ſame Plane, GN and CF, which, if infinitely produced on th Sides, would never meet. *Fig.* 2.

6. A Surface has only Length and Breadth; whoſe rms are Lines.

7. A plain Surface lies evenly betwixt its Terms, d is produced by the Motion of a right Line.

8. A Body is a Magnitude long, broad and deep; oſe Terms are Surfaces. *A Body therefore has*

three Dimenſions, a Surface two, a Line one, and Point none.

Fig. 3. 9. A plain Angle is the Inclination of two Li AB and AC to one another in the ſame Pla which touch each other in the Point A, but ſo not to make one right Line.

10. The Lines forming an Angle are called Sides, or Legs.

11. The Vertex, or Top, of the Angle is whe the Legs meet, as in the Point A.

12. A right-lined Angle is made of ſtreight Line a curvilinear Angle of crooked Lines, as B; and mixed one of a ſtreight Line and a crooked one, as

Fig. 3. 13. A ſingle Angle is deſigned by one Letter at t Top, as X; but when there are more at one Poi they are expreſſed by three Letters, BAF; where that at the Top of the Angle is placed in the Middl

14. Angles are equal, if, when the Tops are la one on another, the Sides of the one exactly fall up the Sides of the other; but it is not required t Sides ſhould be of the ſame Length.

Fig. 4. 15. When a right Line, AD, ſtanding on anoth right Line CB, makes on both Sides thereof the A gles CAD, and DAB, equal between themſelv each of theſe Angles is called a right one; and t right Line AD ſtanding on the other, is called Perpendicular Line.

16. An obtuſe Angle, CAX, is greater than right one.

17. An acute Angle, XAB, is leſs than a right o

Theſe two Angles are often called oblique Angles.

18. A Triangle is a plain Surface contained u der three right Lines.

Fig. 5. 19. An equilateral Triangle has all its three Sid equal.

Fig. 6. 20. An Iſoceles, or Equicrural Triangle has on two Sides equal.

Fig. 7. 21. A ſcalene Triangle, has all its three Sides u equal.

Fig. 8. 22. A right angled Triangle has one right Angl as B.

The Side AC, *oppoſite to the right Angle* B, *is ca* *the Hypothenuſe, the longeſt Side* C B, *of the other* *o, including the right Angle, is called the Baſe and* *third Side* B A *the Perpendicular, or Cathetus.*

23. An obtuſe angled Triangle has one obtuſe An- *Fig.* 7. , as X.

24. An acute angled Triangle has all its three An- *Fig.* 5. s acute, as A, B and C.

25. The Height of a plain Triangle, is the Length *Fig.* 6, a Line A D, let fall perpendicular from any of its and 7. gles upon the Side oppoſite to that Angle from ence it falls, and it may either fall within, or thout the Triangle.

26. A Circle is a plain Figure contained under *Fig.* 9. Line called the Circumference, or Periphery, to ich all right Lines, drawn from a certain Point C lled the Center) are equal.

27. The Diameter is a right Line A B, drawn ough the Center, and terminated on both Sides the Circumference; and it divides the Circle two equal Parts.

8. The Semidiameter, or Radius, is a right Line), drawn from the Center to the Circumference.

9. A Quadrant, A E, is the Quarter of a Circle e by the Radius E C, perpendicular to the Dia- er A B, at the Center C, cutting the Periphery he Middle at E.

0. A Chord or Subtenſe, A G, of an Arch is a t Line cutting the Circle into two unequal Parts, is leſs than the Diameter.

1. A Segment, of a Circle, is a Figure included wixt the Chord and that Arch of the Periphery off by the Chord; and is either greater or leſs a Semicircle, as A E G, or A F G.

2. A Sector is a Part of a Circle contained under Semidiameters C D, and C B, and an Arch B D, ch is the Meaſure of the Angle B C D, intercept- etwixt the Semidiameters.

ll the Angles of Sectors are Angles at the Center.

3. An Angle in the Segment of a Circle, F A G, ntained under the Lines F A and A G, drawn one Point A of the Circumference to the Ends e Segment, F and G.

34. An Angle of a Segment, as A, is containe under the right Line A F, and Part of the Ci cumference A F.

35. An Angle, G A B, is ſaid to ſtand on the Ci cumference G B, as being oppoſite to it.

The Circumference is divided into 360 *Degrees, t Semicircumference into* 180, *and the Quadrant into* 9

Fig. 10. 36. A Rectangle, A B C E, or a right angled P rallelogram, has four right Angles, and conſequent equi-angled and its oppoſite Sides equal.

Fig. 11. 37. A Square, A B C D, has equal Sides, and right angled, and conſequently equi-angled.

Fig. 12. 38. A Rhombus, A B C D, is equilateral, but n equi-angled.

Fig. 13. 39. A Rhomboides, B Z, has its oppoſite Sides a Angles equal, but is neither equilateral, nor equi-a gled.

Fig. 12, and 13. 40. The Heighth of a Rhombus, or Rhomboide is a Line B E, or C F, let fall perpendicular fro any Angle upon the Side oppoſite to that Angle, a may be either within or without the Figures.

All other Figures that differ from theſe abovem tioned, are called Trapezia.

Fig. 10. 41. The Diameter, or Diagonal, of a Parallel gram, and every Quadrilateral Figure, is a rig Line E B, drawn through the oppoſite Angles, a divides the Figure into two Triangles.

Fig. 14. 42. The external Angle of a right Lined Figu ariſes without the Figure, when the Side is prod ced. Such are, A E B, B F C, C H E.

Every Figure therefore has ſo many external Ang as it has Sides, or internal Angles.

All other Figures, that have more than Four Sid are called Polygons, whether regular, or irregular.

Polygons are named according to the Number of th Sides, as a Pentagon is that which has five Sides, Hexagon ſix, an Heptagon ſeven, an Octogon eight, &

Fig. 15. 43. A Parallelepiped is a Body bounded by ſix P rallelograms, whereof the two oppoſite ones are way. equal and parallel.

Fig. 16. 44. A Cylinder is a Body made by the Rotation o right angled Parallelogram, A B C D, round one o Sides, till it end where it began.

45. A Pyramid is a Body E F, bounded by ſeveral *Fig.* 17. Triangles, E C A, E A F, E A D, E D B, making ne Surface for the Baſe F, meet all in one Point E.

46. A Cone is a Body made by the Rotation of a *Fig.* 18. ight angled Triangle, A B C round a Circle, the ngular Point of the right Angle being fix'd in the Center.

47. A Priſm is a Bodyhaving two parallel Planes ſi- *Fig.* 19. milar and equal; and the othersParallelograms, as AD.

48. A Sphere is a Body produced by the Motion *Fig.* 20. f a Semicircle B D E round its Diameter B D, which remains immoveable.

49. A Tetrahedon, or Pyramid, is a Body con- *Fig.* 21. ained under four equal and equilateral Triangles.

50. An Hexahedron, or Cube, is a Body (like a *Fig.* 22. Dice) bounded by ſix Squares.

51. An Octahedron, is a Body bounded by eight *Fig.* 23. qual and equilateral Triangles.

52. A Dodecahedron, is a Solid contained under *Fig.* 24. welve equal and equilateral Pentagons.

53. An Icoſahedron, is a Body contained under *Fig.* 25. wenty equal and equilateral Triangles.

Theſe laſt five, are called the five regular Bodies.

GEOMETRICAL PROBLEMS.

PROBLEM I.

To divide a given Line A B, *into two equal Parts.* *Fig.* I.

FROM the Centers A and B, deſcribe two Circles, with the Radius more than half the Line, utting one another in C and D, and ſo draw the right ine C D. The ſame biſects the given Line A B.

Hence a Perpendicular may be raiſed upon the Middle f a given Line, or one may be let fall from the Points C *and* D, *to the given Line.*

PROB. II.

rom a given Point A, *in a given right Line* C B, *to raiſe a Perpendicular* A D. *Fig.* 4.

From the Point A, take the equal Lines A C, and AB, from the Centers C and B, deſcribe two Circles cutting one another in D. The Line drawn from A to D, will be the Perpendicular.

PROB. III.

Fig. 26. *From a given Point* A, *without a right Line* L X, *to let fall a Perpendicular to that Line.*

From the Center A, deſcribe a Circle, cutting the given Line L X in C I. Biſect the Line C I, with the Line AB. This AB, is the Perpendicular.

PROB. IV.

Fig. 27. *To raiſe a Perpendicular upon the End of a given Line* AB.

Upon any Point out of the given Line, as at C, deſcribe a Circle paſſing through the Point B, from which the Perpendicular is to be raiſed. From the Point A, where the Circle cuts the given Line, draw the Diameter A C D. From the Point D, draw the Line D B, which will be the Perpendicular.

Theſe three laſt Problems *are eaſily done by the Help of a Square.*

PROB. V.

Fig. 2. *Thro' a given Point* F, *to draw a Parallel to a given right Line* AB.

Take any Point in the given Line, as X, with the Diſtance X F, deſcribe a Semicircle. Take the Arch C G equal to the Arch F N. Thro' the Points C and F, draw the Line C F, which will be the Parallel.

PROB. VI.

Fig. 28. *To divide an Angle,* B A C, *into two equal Parts.*

Take from the Sides of the Angle two equal Lines A E and A D; then from the Centers E and D, deſcribe two equal Circles, cutting one another in F and draw the Line F A, This biſects the Angle.

Hence, an Angle may be divided into 4, 8, 16. &c. *equal Angles,* viz. *by biſecting each Part again.*

PROB. VII.

Fig. 29. *At a given Point* B, *in a right Line, to make an Angle equal to a given one* A.

From the Point A, with any Radius, deſcribe an Arch N M, and from the Point B, with the ſame Radius, deſcribe the Arch F G. Take the Arch F X equal to the Arch N M. Then thro' the Points B and X, draw a right Line, the Angle F B X, will be equal to the given one A.

A Proportion is of great Uſe, I have given the following Sketch.

QUANTITIES are in Proportion to one another, when the firſt Term contains the ſecond, as often as the third contains the fourth, or when the firſt is as often contained in the ſecond as the third is in the fourth. So 12 : 6 : 4 : 2. And 3 : 9 : 4 : 12 are alike; becauſe in the former Example 6 and 2, are contained twice in the reſpective Antecedents, and ſo the Proportion of the Antecedents is double to the Conſequents. In the other Example the Proportions are alſo alike, becauſe the Conſequents 9 and 12, contain their reſpective Antecedents three times, and ſo the Proportion of the Antecedents to the Conſequents is ſubtriple.

See, in ſhort, all the different Ways how like Proportions may be changed and ordered among themſelves, ſo that the emerging Proportion on both Sides may be ſtill alike.

Let it be	16 : 8 : : 4 : 2
Then it will be by Alternating.	16 : 4 : : 8 : 2
Inverting	8 : 16 : : 2 : 4
Compounding	16 + 8 : 8 : : 4 + 2 : 2
Dividing	16 — 8 : 8 : : 4 — 2 : 2
Converting	16 : 16 + 8 : : 4 : 4 + 2 16 : 16 — 8 : : 4 : 4 — 2 16 + 8 : : 8 : : 4 + 2 : 2
Comparing orderly	16 : 8 : : 4 : 2 & 8 : 4 : : 2 : 1
Then	16 : 4 : : 4 : 1
Comparing diſorderly	16 : 8 : 1 : 4 : 2 & 8 : 32 : : 1 : 4
Then	16 : 32 : : 1 : 2

A Geometrical Conſtruction of Continual Proportion.

Fig. 52. THIS Scheme is compoſed of Iſoceles Triangles. Let d x y be a given Triangle: on one of its longeſt Sides, d x, make an Iſoceles Triangle x d n, which is ſimilar to the given one, becauſe the Angle, x d n, is common to both: Then on d n make the Triangle b d n, and on b d make the Triangle b d s, &c.

Therefore, as the longeſt Side of the one, is the ſhorteſt Side of the other, and they are all ſimilar. The Proportion will be, as d y : d x : : d x : x n : : x y : n b : : n b : b s : : b s : st : : st : rt : : rt : r z, &c.

PROB. VIII.

Fig. 31. *To divide a Line* A B, *according to a given Proportion, as* F I *to* I L.

Draw an infinite Line A Z, from which take A Q, Q R equal to F I and I L, and from R draw R B: parallel to this draw Q C. The Thing is done.

PROB. IX.

Fig. 32. *To divide a Line* A B *in like manner, as another given one* A I *is divided in* F *and* C.

Let the Line I B join the Extremities of the two Lines, draw Parallels to this from the Points F and C, meeting the Line A B, to be cut in L and Q. The Thing is done.

PROB. X.

Fig. 33. *Two Lines* A B, *and* B C, *being given to find a third Proportional.*

Draw A C, and from B A produc'd, take A F equal to B C; through F draw the infinite Line F x parallel to A C: And let B C be produc'd, till it meet x L in L. A B is to B C, as B C to C L.

PROB. XI.

Fig. 34. *Three Lines* A B, B C, *and* A F, *being given to find a fourth Proportional.*

Make at Pleaſure the Angle F A C, and take upon A C the Line A B and B C, draw the Line B F; to which draw C Z parallel, let A F produced to A L, meet C Z. A B is to B C, as A F is to F L.

PROB. XII.

Two Lines A C *and* C B, *being given to find a Mean Proportional.* Fig. 35.

Biſect the compound Line A B in O, and from the Center O, deſcribe a Semicircle; from C erect a perpendicular meeting the Circumference in F. A C is to C F, as C F is to C B.

Hence all Triangles may be turned into Squares, by finding a mean Proportional between the Baſe and Half the Perpendicular.

PROB. XIII.

To cut a right Line AB, *in extream and mean Proportion.* Fig. 36.

From A erect a Perpendicular A F equal to A B, biſect A F in X: draw X B; from F A produced, cut off X I equal to X B. The Thing is done. For AB is to A C, as A C to C B.

Hence all Squares, Parallelograms, and Triangles may be divided into extream and mean Proportion.

PROB. XIV.

Upon a given Line A B, *to make an equilateral Triangle.* Fig. 6.

From the Center A, with the Diſtance A B, deſcribe an Arch; and from the Center B, with the ſame Diſtance, deſcribe another Arch, cutting the former in C; from which Point draw the Lines C A and C B. The Triangle A B C, will be an equilateral one.

PROB. XV.

To make a Triangle of three given Lines A B, C G *and* D F, *whereof any two of them muſt be greater than the third.* Fig. 37

Let D F, one of the given Lines be taken, and one of its Extremities for the Center, with the Diſtance of the other given Line C G, deſcribe an Arch; from the other End of D F, with the Diſtance of the third Line A B, deſcribe another Arch cutting the former in E; then draw the Lines D E and E F, and they will form the Triangle required.

Hence a Triangle may be made equal to any given one.

PROB. XVI.

Fig. 38. *To divide a Triangle* ABC, *into any Number of equal Parts*, e. g. *into five.*

Firſt, Divide the longeſt Side B C into five equal Parts, join the fifth Part D to A; divide the longeſt of the other two Sides into four equal Parts, join the fourth Part G to D; divide the remaining Part D G, into three equal Parts; join F to G; divide the Remainder G C, into two equal Parts, and join E to F. The Triangle A B C is divided into five equal Triangles, *viz.* ABD, ADG, GDF, GFE, and EFC.

PROB. XVII.

Fig. 39. *To divide a Triangle* A B C, *into any Number of equal Parts, by right Lines drawn from a given Point* D *in one Side*, e. g. *into three.*

Divide the Side A B into three equal Parts in the Points E and F, and join C D; draw to it thro' the Points of Diviſion E and F, the Parallels E G and F H, which will give in the Sides A C and B C, the Points G and H: thro' which, and the given Point D, draw the Lines D G and D H, which will divide the Triangle into three equal Parts.

If the given Point D be in the Middle of the Side A B, the Triangle can be divided into ſix equal Parts, by dividing each of the two Sides A C and B C into three equal Parts at the Points L, G, M and H, and joining the Lines DL, DG, DH, DM, *&c.*

PROB. XVIII.

Fig. 40. *To divide a Triangle into three equal Parts, by Lines perpendicular to the Side* B C.

Draw A D perpendicular to B C from its oppoſite Angle, and divide either of its Segments, *e. g.* B D into three equal Parts at E and F; take in B C a mean

Proportional B G, betwixt B C and B F of the Segment B D, and take B D a mean Proportional betwixt BC and ⅔ BE of the Segment B D; and draw from G and H upon B C the Perpendiculars G I and H K, which will divide the Triangle ABC into three equal Parts.

The Triangle ABC may, in like manner, be divided into equal Parts by right Lines, making an Angle B A D with the Side B C equal to the given one B, and having found, as before, G and H, draw, thro' theſe Points, the Lines G I and H K parallel to A D.

PROB. XIX.

To cut off from the Triangle A B C, *a Triangle at* B *equal to a given one* B D E. *Fig.* 41.

Make at the Point D the Angle B D F equal to the given Angle A B C, draw E F parallel to B D and F B to E D, take B G equal to D F and B H equal to B D: draw G H, which will cut off the Triangle B G H equal to the given one B D E.

Quadrilaterals may be eaſily divided by one who underſtands the Diviſion of Triangles, tho' the Diviſion of Triangles depends, in ſeveral Caſes, upon the Diviſion of Quadrilateral Figures, and principally Trapezia; as, when a Triangle is to be divided into more than two equal Parts.

PROB. XX.

To make a ſcalene Triangle A B C, *equal to a right angled Triangle.* *Fig.* 42.

Draw B D parallel to the Baſe A C, from A erect the Perpendicular A D of the ſame Height with the Parallel, and draw C D. The Triangle A D C is a right angled Triangle, and equal to the Triangle A B C.

PROB. XXI.

To make a Scalene Triangle A B C *equal to an Iſoceles.* *Fig.* 43.

Draw B D parallel to A C, make A E equal to E C; from E raiſe the Perpendicular E F and draw A F and CF. The Iſoceles Triangle E F C is equal to the Scalene A B C.

PROB.

PROB. XXII.

Fig. 44. *To make a Scalene Triangle* A B C *equal to an equilateral one.*

Upon A C make an equilateral Triangle A D C; upon A D deſcribe a Semicircle A F D, draw B E parallel to A C. From E raiſe the Perpendicular EF cutting the Semicircle in F, draw A F upon which make the equilateral Triangle A G F, which is equal to the ſcalene one.

Fig. 45.

PROB. XXIII.

To find the Center of a given Circle.

Draw a Line BC in the Circle at Pleaſure, which biſect in Q; thro' I, draw the Perpendicular L F, which biſect in A. A will be the Center.

Fig. 46.

PROB. XXIV.

To deſcribe a Circle, whoſe Area ſhall be double to that of a given one.

Divide the Circle A B C D into four equal Parts, take the Diſtance AB and put it from E to F, and with the Diſtance EF deſcribe a Circle, whoſe Area will be double to the Area of the given Circle.

Hence, the Area of a Circle may be made three, or four times, &c. bigger than the given one.

Fig. 47.

PROB. XXV.

To deſcribe a Circle that ſhall paſs thro' three Points, not lying in a ſtrait Line, as the Points A, B *and* C.

Join the Points B A and B C with right Lines; divide the Line A B into two equal Parts, and draw the Line XY. Alſo divide the Line B C into two equal Parts, and draw the Line Z Y, and at Y the Point of Interſection will be the Center of the Circle, which will paſs thro' the Points A, B and C.

Hence, any Triangle may be inſcribed in a Circle, by biſecting its Legs AB and BC.

Hence, one Arch of a Circle may be perfected, by drawing two Lines A B and B C *within the given Arch, and biſecting them.*

Hence, the Center of a Circle may be found, by drawing within the Circles two Lines A B *and* B C, *and biſecting them.*

P R O B.

PROB. XXVI.

To make an Oval. *Fig.* 48.

Draw a Line, and thereupon deſcribe a Circle, BAC; from C deſcribe another Circle CBD thro' B: From the Point of Interſection E, draw the Lines EH and EG thro' the Centers B and C; from E iththe Diſtance EG deſcribe an Arch from G to H, nd from the Point F with the ſame Diſtance deſcribe n Arch from I to K, and ſo the Oval is finiſhed.

PROB. XXVII.

o make a right lined Triangle equal to a given Circle. *Fig.* 49.

Divide the Diameter AB of the given Circle into even equal Parts; erect a Perpendicular on B thrice s long as the Diameter AB, and a ſeventh Part o-er as BC: Draw a right Line from C to D the enter of the given Circle, and the Triangle DCB, that which was to be done.

Hence the Circumference of a Circle may be known by aving the Diameter given.

PROB. XXVIII.

rom a Point C *given in the Circumference to draw a Tangent* XY. *Fig.* 46.

From the Center E thro' the given Point C draw e Line EZ, ſo that CZ may be equal to EC, bi-ct EZ in C with the Line RS, which will be the angent required.

PROB. XXIX.

To draw a Spiral Line. *Fig.* 50.

Upon a ſtreight Line XY deſcribe an Arch ABC; om the Point C with the Diſtance BC deſcribe an-her Arch touching the ſame Line in D; from the ſt Center A deſcribe another Arch from D that ſhall t the Line in E: Return to the Point B, and from as the Center with the Diſtance, deſcribe another ch to F. Take A as the Center, and deſcribe an ch from F to G; and ſo on to Infinity.

PROB. XXX.

To make a Square ABCD. *Fig.* 11.

Upon B raiſe a Perpendicular B C of the ſame Height, with the ſame Diſtance make two Arches from A and C, interſecting each other at D and join D A and A C by right Lines, which will compleat the Square A B C D.

PROB. XXXI.

Fig. 51. *To make a Square* D F, *equal to a given Circle.*

Divide the Diameter A B into ſeven equal parts, double the Diameter, and add a ſeventh part of itſelf to it, as, A C; divide the firſt Diameter A B equally in D, and divide the Line D C equally in E, with the Diſtance D E deſcribe an Arch D F C. Upon E raiſe the Perpendicular E F touching the Circle; this Line E F will be a Side of the Square required.

PROB. XXXII.

Fig. 53. *To inſcribe in a Circle a Triangle* D, *having equa Angles with a given one* X.

Let the Line E F touch the Circle in D, mak E D G equal to Angle C, and F D H equal to B, an join G H. The Thing is done.

A right lined Figure is ſaid to be inſcribed in a Circle, when all the angular Points of the Figure touch th Circle's Periphery.

PROB. XXXIII.

Fig. 54. *To inſcribe a Circle in a Triangle.*

Biſect the two Angles with the Lines C A and E meeting together in A, which will be the Center o the Circle to be inſcribed.

PROB. XXXIV.

Fig. 55. *To inſcribe a Square in, and circumſcribe one about Circle.*

Draw the Diameters B D and C E cutting one an other perpendicularly. The Lines joining th Terms of theſe, inſcribe a Square in a Circle. Th draw the four Tangents touching the Circle in B, C D and E meeting together in I, F, G and H. Th Figure I F G H is a Square circumſcribed about Circle.

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PROB. XXXV.

To make a Square two, three or four times greater than it is. Fig. 56.

Extend the Side A B, take the Diſtance B D, and ſet upon the extended Side A B, from B to E, and it will be the Side of a Square double to the given one A B C D, and ſo on.

PROB. XXXVI.

Three or more Squares being given, whoſe Sides are A B, B C *and* C E, *to make one Square equal to them all.* Fig. 57.

Make a right Angle Triangle F B Z, having indefinite Sides, and upon the Sides of it tranfer the Lines A B and B C, then join A C. The Square of A C will be equal to the Squares A B and C B together. Then transfer A C from B to X, and C E, the third given Side, from B to E, and join E X, the Square of E X ſhall be equal to the three given Squares A B, B C and C E.

PROB. XXXVII.

To make a Square equal to an oblique angled Parallelogram. Fig. 58.

From the Points A and B let fall the Perpendiculars on C D, to AB add B X equal to B F, and divide AX equally in G, and with the Diſtance A G deſcribe a Semicircle A S X. From B erect a Perpendicular touching the Semicircle in S, which will be the Side of the Square required.

PROB. XXXVIII.

To make a Parallelogram with an Angle in it equal to a given one O, *and equal to a given Triangle* A B C. Fig. 59.

Biſect the Baſe A B in F, thro' C draw C X parallel to A B, make the Angle B A L equal to the given one O, and draw F I parallel to A L. A L I F will be the Parallelogram required.

PROB. XXXIX.

To inſcribe a Parallelogram in a Circle, whoſe Length is to the Breadth, as five to two. Fig. 60.

Divide the Diameter X Y into five equal Parts; ſet off two of them from Y to Z, from X thro' Z draw X Z S the Length of the Parallelogram, and from S draw the Breadth of the Parallelogram required.

Hence a Parallelogram may be inſcribed in a Semicircle, whoſe Length is to its Breadth, as five to two.

PROB. XL.

Fig. 61. *To reduce a Square to an equilateral Triangle.*

Upon d z the Side of the Square, take z x at pleaſure, and with that Diſtance deſcribe the Arch x o n, upon x z make the equilateral Triangle x o z. Set off one half of its perpendicular Altitude from z to y, biſect x y in t, on which deſcribe the Semicircle x m y; m z, is a mean Proportional betwixt x z, the Baſe of the equilateral Triangle x o z, and half its Perpendicular z y. The Square of m z is equal to the equilateral Triangle x o z draw o m, parallel to which draw B h till it meet z o produced to h, and on z h make an equilateral Triangle p h z, which will be equal to the given Square. For the Triangles z o m and z B h are ſimilar, by Conſtruction; then, as z m : z o : : z B : z h : that is as the mean Proportional z m is to the Side of its correſpondent equilateral Triangle x o z, ſo is the mean Proportional z B, the Side of the given Square, to the Side of the equilateral Triangle p h z, which is epual to the given Square.

Hence, all regular and irregular Polygons, Trapeziums, Parallelograms and Triangles may be reduced to equilateral Triangles, or to any right lined Triangle, if the Angles are given ſeverally; firſt by reducing them to a Square, and then finding a Triangle equal to that Square.

PROB. XLI.

Fig. 62. *To make a Square equal to a Triangle whoſe Angles are all known.*

Upon the Side C D, make a Triangle C o D, including the Angles of a given Triangle, ſet off half its Perpendicular r o from C to p, on D p, deſcribe a Semia Semicircle D o p, then the Square of C S, will be equal to the Area of the Triangle D O C. Draw o s, parallel to which from B draw B n till it meet O C produced to n: From n parallel to O D draw n t meeting D C produced to t. Set off half the Perpendicular n m of the Triangle t n c, from C to L: From m the Center deſcribe a Semicircle. The Triangle t n c will be equal to the Square, and the Triangles are ſimilar.

Hence, a Triangle whoſe Area and Angles are given, may be geometrically conſtructed, if a Side of a Square be found equal to the given Triangle.

PROB. XLII.

Upon a given Line R S, *to deſcribe a Polygon like, and in like manner poſited to a given one,* B Q. *Fig.* 63.

Reſolve the given Polygon B Q into Triangles, upon the given Line R S make the Angles R and O equal to the Angles A and B, (by Problem 7.) the Sides will then meet together in X: Upon X S make the Angles V and I equal to the Angles T and C: The Sides will then meet together in Z. The Thing is done.

Hence, is derived the Method of making Maps, or Charts, whether Geographical or Chorographical, or thoſe which Surveyors of Land make; and of framing Ichnographical Delineations of Fields, Buildings, and Countries: For they are only but the Reduction of great Figures unto like Figures of a ſmaller Compaſs, and is performed by this Problem.

PROB. XLIII.

Upon a given Line to deſcribe a regular Pentagon. *Fig.* 64.

Make the given Line A B Radius, and upon each End deſcribe a Circle, and thro' the Points X an G of Interſection draw the Line GEX. From G, with the ſame Radius, deſcribe an Arch H E D, lay a Ruler to the Points D and E, and mark where it croſſes the other Circle, as F, alſo to to the Points H and E, and mark where it croſſes the other Circle, as C. From the Points F and C, with the ſame Radius, deſcribe two Arches cutting one another in K, join the Points AF, KC, F K, and C B with right Lines, and they will form the Pentagon required.

PROB. XLIV.

Fig. 65. *To deſcribe a regular Pentagon in a Circle.*

Draw the Diameters E D and B F cutting one another perpendicularly in A. Biſect the Radius AD in C. From the Center C thro' B deſcribe an Arch, including the Diameters E D in G. The Line D G is the Side of a Pentagon, and H G of a Decagon.

Hence a Circle may be divided into five, ten, twenty, &c. *equal Parts.*

PROB. XLV.

Fig. 66. *To reduce a Triangle* ABC *to a Pentagon* DEFGC.

Reduce the Triangle ABC to a Square EXDC, with any Diſtance on BC, as C y, deſcribe the Circle k z C, in which make a Triangle k y C, and ſet off one half of its Perpendiecular L y, from C to T on k T, from S deſcribe the Semicircle k y T. So the Square of n C, will be equal to the Triangle k y C, draw y n; parallel to which draw x m till it meet BC produced, and I m, parallel to k y, then the Triangle I m C, will be the Triangle of a Pentagon, and equal to the Triangle ABC; from B let fall the Perpendicular B y, and ſet off the one tenth of B y from C to u; then from h with the Diſtance I h, deſcribe the Semicircle I n u. Draw F i parallel to y n, and F y parallel to y k; with the Diſtance F C, deſcribe a Circle C G X, and with the Diſtance q C deſcribe the Pentagon q C O P G, which was to be done.

Hence, any right lined Figure can be reduced to any regular Polygon, and any regular Polygon to any other Polygon, or to any right lined Triangle, if the Angles are given.

PROB. XLVI.

Fig. 67. *To reduce a Pentagon* ABCDE, *to an irregular Polygon.*

Reduce the Pentagon ABCDE, to the Square GE. Draw a Polygon a b c o d E ſimilar to that required, and reduce it to a Square, (by *Euclid.* l. ii. Prop.

rop. 14) of which make N E one Side, draw N O
nd H e parallel to it. Draw e P, c M, M h, h t
arallel to od, oc, c b, ba. So the Polygon t h m e PEt,
ill be equal to the Square G E, which is e-
ual to the Pentagon, for it is as N E : H E : : o E :
E, (by *Euclid*. l. vi. Prop. 4.) (by Prop. 19.) as the
quare of N E : H E : : o E : e E.

Hence, one irregular Polygon can be reduced to an-
ther irregular one.

Hence alſo, all right lined Figures can be divided
nto extream and mean Proportion; for if any Side of
be Figure be divided into extream and mean Propor-
ion, and on each Segment be drawn a Polygon ſimilar
that given, theſe Polygons will be directly as their
ides.

P R O B. XLVII.

Upon a given Line to raiſe a regular Hexagon. *Fig.* 68.

Make on B C an equilateral Triangle C A B;
rom the Center A thro' B and C, deſcribe a Circle.
his will contain a Hexagon upon the given Line
B.

P R O B. XLVIII.

To inſcribe a regular Hexagon in a given Circle. *Fig.* 69.

Draw the Diameter F A B; from the Center B,
hro' A deſcribe a Circle cutting the given one in
and D; alſo from F thro' A, a Circle cutting the
iven one in E and G. The ſix Points B, C, E, F,
and D, joined by right Lines, will give the
exagon required.

Hence, a Circle can be divided into twelve, twenty-
our, forty-eight, &c. *equal Parts.*

The Side of an Hexagon inſcribed in a Circle, or
Chord of ſixty Degrees, is equal to the Radius, and
onſequently the Sign of thirty Degrees is equal to half
he Radius.

An Angle of a regular Hexagon is four thirds of a
ight Angle, as conſiſting of two Angles of an equila-
eral Triangle, each of which makes two thirds of a
ight Angle.

An equilateral Triangle is eaſily inſcribed in a Cir-
le, by drawing the Diameter F B, *and deſcribing from*

the Center B *thro'* A *the Arch* CAD. *The Points* CFD *joined with right Lines, will give the Triangle ſought.*

PROB. XLIX.

To inſcribe a regular Quindecagon in a Circle.

Fig. 70. Inſcribe in the Circle AC the Side of a Pentagon, and AD the Side of an equilateral Triangle, biſect the Arch CD in E, CE is the Side of the Quindexagon ſought.

For if the Circumference be ſuppoſed to be fifteen, the Arch AC will be three, and the Arch AD five, therefore the Arch CD two, and conſequently CE one.

Hence innumerable regular Figures may be inſcribed in Circle; for if AC, *and* AD *the Sides of the two regular Figures be inſcribed in a Circle, the Difference of the Arches* CD, *will contain ſo many Sides of a new regular Figure as the Units, are whereby the Denominators of the former differ one from another. But the Denominators of the new Figure is had, if the Denominators of the former be multiplied one by another. As, if* AD *be the Side of a Square, and* AC *of a Decagon, the Difference of the Denominators is ſix. Therefore the Arch* CD *contains ſix Sides of a new Figure. But the new Figure is of forty Sides. For the Denominators* 4 *and* 10, *multiplied the one by the other make* 40.

PROB. L.

Fig. 70. *To inſcribe regular Figures* 7, 9, 11, 13, 17 *Sides* &c. *in a Circle.*

Divide the whole Circumference, *viz.* 360, by the Denominator of the Polygon to be inſcribed, *e. g.* a nineſided Figure. Make at the Center the Angle AKG of ſo many Degrees as there are Units in the Quotient, which will be 40; AK will be the Side of the nine angled Figure required to be inſcribed in the Circle.

PROB. LI.

Fig. 70. *Upon a given right Line any regular Figure whatever may be deſcribed by the following Table.*

A right Angle is to the Angle of the Figure.

					Difference
In	A Pentagon	as	5	to	6—1
	An Hexagon		3		4—1
	An Heptagon		7		10—3
	An Octagon		2		3—1
	A Nonagon		9		14—5
	A Decagon		5		8—3
	An Undecagon		11		18—7
	A Duodecagon		3		5—2

Deſcribe a regular Heptagon on the given Line XB. From the Center X, with the Diſtance XB, deſcribe a Circle, from which cut the Quadrant BO. See in the Table the Proportion of a right Angle to the Angle of an Heptagon, which will be as 7 to 10, and the Difference 3. Therefore divide the Quadrant into 7 equal Arches, ſo many of which add to it from O to N as the Difference has Units. Thro' the three Points B, X and N deſcribe a Circle; this contains an Heptagon on the given right Line XB.

PROB. LII.

To make a Cylinder equal to a Parallelepiped of the ſame Height.

Fig. 16, & 15.

Upon the Baſe of the Cylinder CBAD make a Square ABCD, upon which raiſe four Perpendiculars of the Height of the Cylinder AB, as E, F, G and H, and join EF, EH, FG and GH by right Lines, which will make the Parallelepiped required.

PROB. LIII.

To make a Parallelepiped equal to a Cylinder.

Fig. 15, & 16.

Make a Circle equal to the Baſe of the Parallelepiped, and raiſe this Circle to the ſame Height with the Parallelepiped, and the Work is done.

PROB. LIV.

To make a Cone equal to a Pyramid of the ſame Height.

Fig. 18, & 17.

Make a Triangle, Square, Pentagon or any other Polygon equal to the Baſe of the Cone CBD, and make it the Baſe of the Pyramid, as, ABCD; raiſe, from the Middle of the Baſe F, a Perpendicular F E of the ſame Height with the Cone B A, then draw from A B C D Lines to the Point E, and the Pyramid is made.

PROB. LV.

Fig. 17, & 18. *To make a Pyramid equal to a Cone.*

Make a Circle equal to the Baſe of the Pyramid, raiſe upon its Center a Perpendicular B A of the ſame Height with the Cylinder, and join the Extremities of the Diameter C D till they meet at the Top of the Perpendicular A, and the Cone will be finiſhed.

PROB. LVI.

Fig. 16, 17, 18. *To make a Priſm or Cylinder equal to a Pyramid or Cone of the ſame Height.*

Make the Baſe of the Cylinder three times as much as it is; upon its Center raiſe a Perpendicular of the ſame Height with the Cylinder, and join the Extremities with the Top of the Perpendicular, and the Pyramid will be made.

In like Manner a Cylinder or Priſm may be changed into a Cone.

PROB. LVII.

To make a Pyramid or Cone equal to a Priſm or Cylinder.

Reduce the Baſe of the Pyramid or Cone to one third of the Baſe of the Priſm or Cylinder, and on that Baſe erect a Pyramid or Cone of the ſame Height with the Priſm or Cylinder given.

PROB. LVIII.

To make a Cube equal to a Parallelepiped.

If the Baſe of the Parallelepiped be a Square, find a mean Proportional between its Height and one Side of the Baſe; this mean Proportional, will be the Meaſure of the Cube required. If the Baſe be a Parallelogram, make it equal to a Square; then go on as at firſt till the Work is finiſhed.

PROB. LIX.

To make a Cube equal to a given Cylinder.

Make a Parallelepiped equal to the given Cylinder. then make a Cube equal to a Parallelepiped.

PROB. LX.

To make a Cube equal to a given one.

Make a Parallelepiped equal to a given Cone, then make a Cube equal to that Parallelepipid.

PROB. LXI.

To make a Cone equal to a Globe. *Fig.* 20.

Make a Circle four times bigger than one of the great Circles of the given Globe, raiſe a Cone upon this Baſe of the ſame Height with the Semidiameter A B of the Globe. The Cone C D E will be equal to the Globe A B D.

PROB. LXII.

To make a Cube equal to a given Globe.

Make a Cone equal to the given Globe, then make a Parallelepiped equal to this Cone, and a Cube equal to this Parallelepiped.

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Plain TRIGONOMETRY.

SECT. I.

Of the Definitions and Affections of Triangles, and of Conſtructing the CANON *of Natural and Artificial* SINES, TANGENTS *and* SECANTS.

DEFINITIONS.

PLAIN TRIGONOMETRY, is the Art whereby, from any three given Parts of a plain Triangle, we find all the reſt.

Fig. 1. *Thus,* e. g. *from two Sides* AB *and* AC, *and an Angle* B, *we find by* TRIGONOMETRY, *the other Angles* B *and* C *with the third Side* BC.

Fig. 2. A *Chord* of an Arch or Angle, is a right Line AB, dividing the whole Circle into two parts, and ſubtends both Segments.

Hence, the greateſt Chord that can be drawn in a Circle, is the Diameter.

Hence alſo, all the Chords of Arches greater than a Semicircle, are leſs than the Diameter.

A

TRIGONOMETRY

Fig. 1

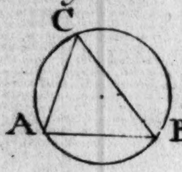


Fig. 2

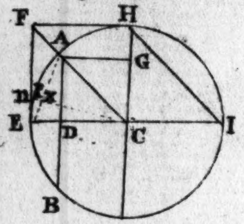


Fig. 3

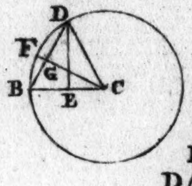


Fig. 4

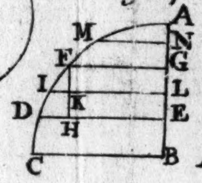


Fig. 4

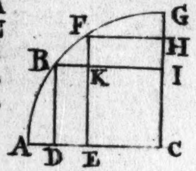


Fig. 6

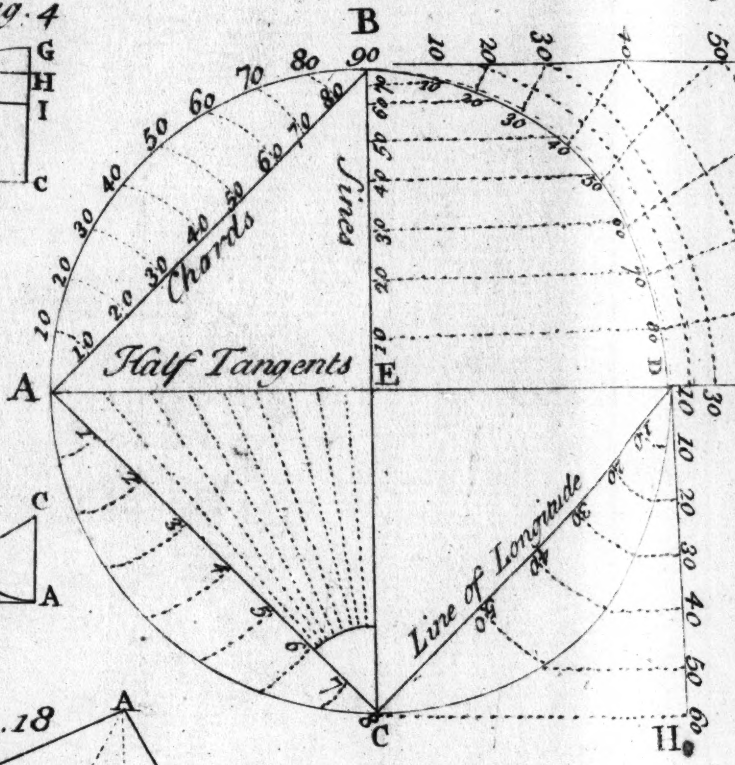


Fig 8

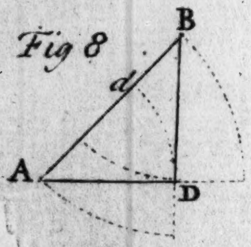


Fig. 9

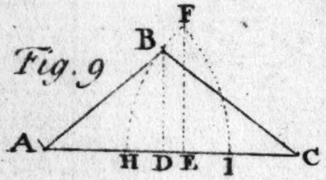


Fig. 10

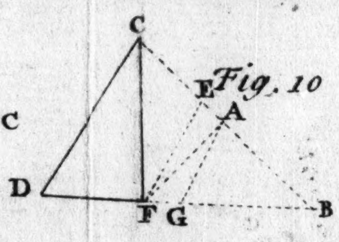


Fig. 11.

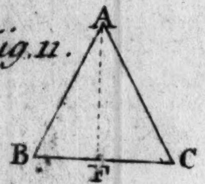


Fig. 12

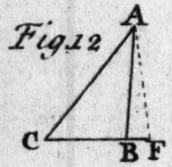


Fig. 12

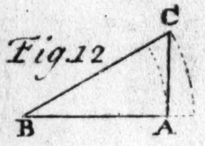


Fig. 13

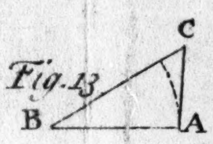


Fig. 14

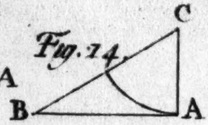


Fig. 5

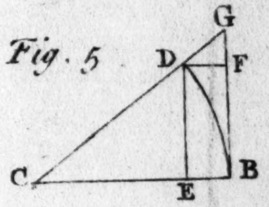


Fig 7

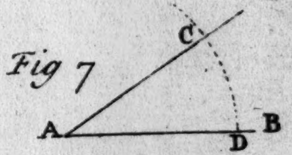


Fig. 15.

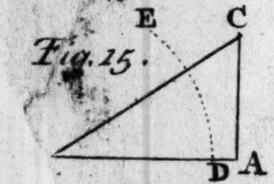


Fig. 16.

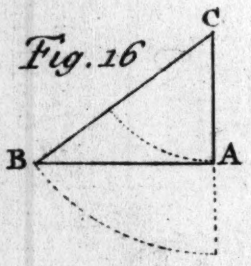


Fig. 17.

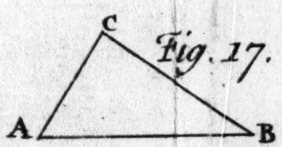


Fig. 18

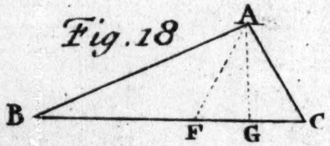


Fig. 24.

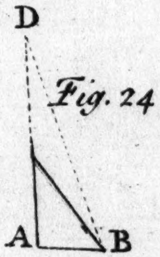


Fig. 25.

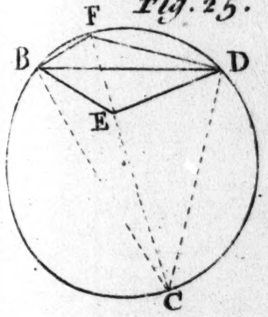
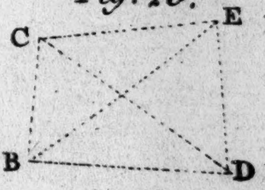


Fig. 26.



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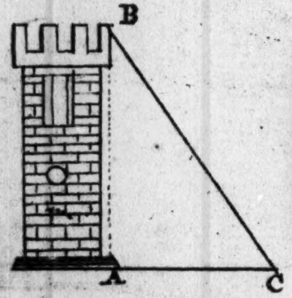


Fig. 20

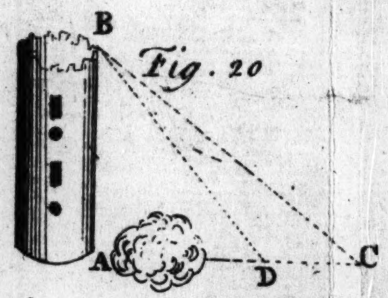


Fig. 21

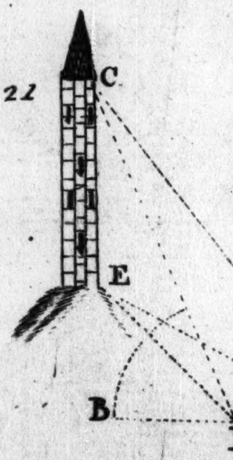
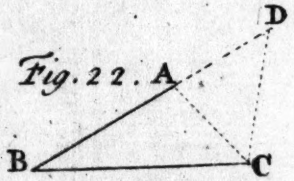


Fig. 22.



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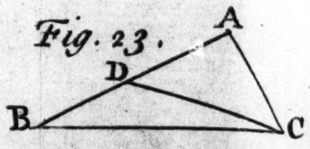
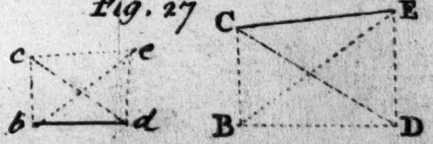


Fig. 27



25

A *right Sine* A D, of the Arch A E or A I, is one half of the Chord A B of the double Arch A E B or A I B.

Hence, the Sine A D *is perpendicular to the Radius* E C, *conſequently, all Sines ſtanding upon the ſame Radius are parallel between themſelves.*

A *whole Sine,* is the Radius H C, or the Sine of the Quadrant H E.

A *verſed Sine,* is that Part of the Radius E D, or D I, intercepted betwixt the right Sine A D, and the Arch A E or A I.

Hence, the greateſt verſed Sine is the Diameter E I.

Since that the Arch A E *is the Meaſure of the Angle* A C E, *and* A I *is the Meaſure of its contiguous Angle* A C I; *but the Quadrant* H E *is the Meaſure of a right Angle;* A D *will alſo be the right Sine, and* E D *the verſed Sine of the Angles* A C E *and* A C I; *but the whole Sine is the Sine of a right Angle.*

Therefore two Angles, which are adjacent, have the ame Sine. Likewiſe, obtuſe Angles have the ſame Sines, which their Complements have to two right ones.

A *Tangent* of an Arch A E is a right Line, E F ouching the Circumference of the Circle, and is at ight Angles to the Diameter E I, and limited by C, called the *Secant* of the ſame Arch.

F E *is alſo the Tangent, and* F C *the Secant of the ngle* A C E, *and alſo of the Angle* A C I.

Therefore two adjacent Angles have the ſame Tan- ent and Secant.

The *Coſine,* is the Sine A G, the *Cotangent* F H the Tangent, and the *Coſecant* F C is the Secant f the Arch A H, which is the Complement of the ther Arch A E to a Quadrant.

The Complement of an Arch, or Angle, is what it ants of a Quadrant, a Semicircle, or of a whole Cir- e. Thus 20 Degrees is the Complement of 70 Degrees a Quadrant; becauſe 20 Degrees is the Remainder of Degrees ſubtracted from 90 Degrees: Alſo, 50 De- ees, is the Complement of 130 to 180 Degrees, and the Complement of 290 to 360 Degrees.

The Radius C A, with the Sine A D and Coſine C, make a Triangle C A D ſimilar to the Triangle CFE made by the Radius CE, Tangent EF and Secant CF. Likewiſe, the Radius, Cotangent and Coſecant make another Triangle ſimilar to the two former.

Hence, as the Coſine is to the Sine, ſo is the Radius to the Tangent. That is, as CD : AD :: CA : EF.

As the Radius is to the Sine, ſo is the Secant to the Tangent. That is, as CA : AD :: CF : FE.

As the Sine is to the Radius, ſo is the Radius to the Coſecant. That is, as DA : CA :: HC : CF.

As the Tangent is to the Radius, ſo is the Radius to the Cotangent. That is, as FE : EC :: CH : FH.

Therefore the Rectangle, between the Tangent and Cotangent of any Arch, is equal to the Square of the Radius.

When a Triangle is given to be reſolv'd, firſt, we are to conſider, that there is in the Table of *Logarithms*, Sines, Tangents, and Secants, a Triangle exactly ſimilar and equal to that which is to be ſolved, and whoſe Sides are to one another in the ſame Proportion of thoſe of the Triangles propoſed.

Next, we muſt underſtand whatever *Ratio* one Side of the given Triangle has to the other Side about the ſame Angle, conſidered as Lengths eſtimated or numbered by any known Meaſure : As ſuppoſe, Inches, Yards, Miles, *&c.* the very ſame has the two Sides about the ſame Angle, in the Triangles in the Tables, or in the tabular Parts ; which two Things, well underſtood, will lead us into the whole Myſtery of Trigonometrical Calculations.

In eſtimating the Quantity of Sines, &c. *we aſſume Radius for Unity; and determine the Quantity of Sines, Tangents and Secants in Fractions thereof. From* Ptolomy's Almageſt, *we learn, that the Antients divided the Radius into* 60 *Parts, which they called Degrees; and thence determined the Chords in Minutes, Seconds and Thirds; that is, in ſexageſimal Fractions of the Radius, which they likewiſe uſed in reſolving Triangles. The Sines, or half Chords, were firſt uſed by the* Saracens. Regiomontanus, *firſt, with the Ancients,* *divided the Radius into ſixty Degrees, and determined the Sines of ſeveral Degrees in decimal Fractions thereof. But he afterwards found it would be more commodious to aſſume Radius for one; and thus introduced the preſent Method into* Trigonometry. *In common Tables of Sines and Tangents, the Radius is ſuppoſed to be divided into* 10,000,000 *Parts, beyond which we never go in determining the Quantity of Sines and Tangents. Hence, as the Sine of a Hexagon ſubtends the ſixth Part of a Circle, and is equal to the Radius, the Sine of* 30 *Degrees is* 5,000,000.

TRIGONOMETRICAL PROBLEMS.

PROB. I.

The Sine AD *being given, to find the Coſine, or Sine Complement,* AG. *Fig.* 2.

BECAUSE that EC, the Sine of the ſame Arch EH, is perpendicular to HC and AG; the Sine of the Arch AH is perpendicular to the ſame HC; AG will be parallel to DC, and the Angle AGC a right Angle, and ſo AGC will be a right angled Triangle. Wherefore, ſeeing AD and HC are Perpendicular to EC; GC will be equal to AD. If therefore from the Square of the Radius AC be ſubtracted the Square of the Sine AD, or GC, the Remainder will be the Square of the Coſine AG. Whence if the ſquare Root be extracted, it will give the Coſine AG, *e. g.* Let AC be 10,000,000, AD 5,000,000, AG will be 8,660,254, the Sine of 60 Degrees.

PROB. II.

The Sine AD *of the Arch* AE *being given, to find the Sine of half that Arch.* *Fig.* 2.

Find the Chord of the Arch AE; for half of this is its Sine. Thus, *e. g.* DG and AD, as in the preceeding Problem; we ſhall find the Sine of half the Arch AE, or the Sine of 15 Degrees =2,588,190.

PROB. III.

Fig. 3. *The Sine* DG *of the Arch* DF *being given, to find the Sine* DE *of the double Arch* DB.

Since the Angles at E and G are right Angles, and the Angle B is common to both Triangles, BCG and DEB; BC : CG :: BD : DE. Wherefore CG being found by the ſecond Problem, and BD deing double of DG; DE is found by the Rule of Proportion.

Hence, CB : 2CG :: BD : 2DE, *that is, the Radius is double to the Coſine of one half of the Arch* DB, *as the Subtenſe of the Arch* DB *is to the Subtenſe of double the Arch. Alſo, as* CB : 2CG :: | 2BG : 2DE :: | BG : DE :: $\frac{1}{2}$CB : CG. *Wherefore, the Sine of any Arch, and the Sine of its Double being given, the Coſine of the Arch itſelf is given.*

PROB. IV.

Fig. 4. ***The Sines* FG *and* DE *of the Arches* FA *and* DA, *whoſe Difference* DF *is not greater than* 45 *Minutes, being given, to find any intermediate Sine, as* IL.**

To the Difference FD of the Arches, whoſe Sines are given; the Difference of the Arch IF, whoſe Sine is required, and the Difference of the given Sines DH, find a fourth Proportional: This added to the leſs given Sine FG, the Aggregate will be the Sine required.

PROB. V.

The Sines BD *and* EF *of the two Arches* AB *and* AF, *being given, to find the Sine* BF *of the Arch of half the Difference.*

Subtract the leſſer-Sine BD from the greater EF, and the Remainder will be FK. From the given Sines BD and EF find the Coſines BI and FH, by Problem I, ſubtract the leſſer Coſine FH from the greater BI, the Difference will be BK. Extract the ſquare Root from the Sum of the Difference of the Squares, the Remainder will be BF, the half of which is the Sine ſought.

PROB.

PROB. VI.

To find the Sine of 45 Degrees. Fig. 2.

Let HI be a Quadrant of the Circle, then will
CI be a right Angle; conſequently the Triangle,
ctangular: Therefore $HI^2 = HC^2 + CI^2 = HC^2$;
herefore, ſince HC the whole Sine, is 10,000,000;
from $2HC^2$ ſquared 200,000,000,000,000, be
tracted, the ſquare Root 14,142,136, the Chord
I will be the Remainder, whoſe half 7,071,068,
e Sine of 45 Degrees required.

THEOREM VII.

n ſmall Arches, the Sines and Tangents of the ſame Arches are nearly to one another, in a Ratio of Equality. Fig. 5.

The Triangles CED and CBG being equiangu-
r, CE : CB :: ED : BG; but as the Point E ap-
roaches B, EB will vaniſh in reſpect of the Arch
D. Whence CE will become nearly equal to CB.
nd ſo ED will alſo be nearly equal to BG. If
B be leſs than the $\frac{1}{10\,000\,000}$ Part of the Radi-
s, then the Difference between the Sine and the
angent will alſo be leſs than the $\frac{1}{10\,000\,000}$ Part
f the Tangent.

*Since any Arch is leſs than the Tangent, and greater
han its Sine, and the Sine and Tangent of a very ſmall
rch are nearly equal, it follows, that the Arch will be
early equal to its Sine; and ſo in very ſmall Arches it
vill be, as Arch is to Arch, ſo is Sine to Sine.*

PROB. VIII.

he Sine of one Minute or 60″ FG *being given, to find the Sin. of one or more Seconds* MN. Fig. 4.

Since the Arches AM and AF are very ſmall, AMF may be taken for a right Line, without any ſenſible Error in the Decimal Fractions of the Radius, wherein the Sine is expreſſed; that is, the Arches AM and AF, may be taken proportional to their Chords. Wherefore, ſince MN is parallel to FG; AF : FG :: AM : MN. Therefore AF, FG and AM being given, MN is eaſily had.

PROB. IX.

To find the Sine of the Arch of one Minute.

The Subtenſe of 60 Degrees is equal to the Radius ſo the half of the Radius will be the Sine of the Arch of 30 Degrees. Wherefore, the Sine of the Arch o 30 Degrees being given, the Sine of the Arch of 1 Degrees may be found, (by Prob. II.) and ſo on til twelve Biſections being made, we come to an Arch of 52^{2}, 44^{3}, 03^{4}, 45^{5}, whoſe Coſine is nearly equal to the Radius, in which Caſe the Arches are proportional to their Sines: And ſo, as the Arch 52^{2}, 44^{3}, 03^{4}, 45^{5}, is to an Arch of one Minute ſo ſhall the Sine before found be to the Sine of one Minute; and when the Sine of one Minute is found then the Sine and Coſine of two Minutes will be had.

PROB. X.

Fig. 2. *The Sine* AD *of the Arch* AE *being given, to find the Tangent* EF, *and the Secant* FC *of the ſame Arch.*

Becauſe the Sine AD and Tangent EF are perpendicular to the Radius EC, AD will be parallel to EF: Wherefore, as the Coſine DC is to the Sine AD, ſo is the whole Sine to the Tangent EF: Alſo, as the Coſine DC is to the whole Sine AC, ſo is the whole Sine EC to the Secant CF.

PROB. XI.

To conſtruct a Canon of Sines.

The Sines of 30°, 15°, 45° and 36°,. (which we have already ſhewn how to find) being had, we can thence conſtruct a Canon of all the Sines to every Minute, or even a Second For from the Sine of 36°, we find thoſe of 18°, 9°, 4° 30 and 2°15 (by Prob. II.) the Sines of 54°, 72°, 81°, 85° 30 and 87°45′, &c. (by Prob. I.) Again, for the Sine of 45° find the Sine 22°30′, 11°15′, &c. From the Sines of 30° and the Sines of 54° find the Sine of 12°. From the Sine of 12°, find the Sines of 6°, 3°, 1°30, 45 78°, &c. From the Sine of 15° find the Sine of 7°30′ 30° 45′, &c. till you have 120 Sines ſucceeding each other orderly at an Interval of 45 Minutes. Between theſe find the intermediate Sines (by Prob. V.) Thus will the Canon be compleat.

PROB. XII.

To find the Logarithm of any given Number.

The firſt Page of the annexed *Tables of Logarithms* contains all the natural Numbers in their proper Order, from 1 to 100. And againſt every one of theſe Numbers is placed its *Logarithm*, with its *Index* before it.—Thus againſt the Number 28, its Log. is 1.447158; and againſt the Number 89, its Log. is 1.949390: And ſo on for the reſt. In the firſt Column of all the following Pages, under *Num.* the natural Numbers proceed in their due Order, from 100 to 1000. And in the next Column, under 0, againſt every one of theſe Numbers, is the Decimal Part of its Logarithm, without any Index; to which its proper Index muſt be prefixed, according as the natural Number uſed requires, *e. g.* againſt the Number 856, under 0, is 932474; to which if 2 the Index of 856 be prefixed, it will be 2.932474, the compleat Logarithm of 856.

The other five Columns of each Page contain the Logarithms of all Numbers, from 1000 to 10000. Thoſe in the Left-hand Pages are diſtinguiſhed on the Top of the Columns with the Figures 0.1.2.3.4, and thoſe in the Right hand Pages with 5.6.7.8.9. So that to find the Logarithm of any Number between 1,000 and 10,000, as ſuppoſe of 5,468 ſeek for the three firſt Figures, *viz.* 546, in the firſt Column under *Num.* and for the laſt Figure, *viz.* 8 at the Top. Then in the Column under the laſt Figure 8, and over-againſt the three firſt Figures 546, there is 737829; to which if 3, the Index of 5,468, be prefixed, the compleat Logarithm thereof will be 3.737829: and ſo for any other Logarithm of any propoſed Number, not exceeding 10,000. But if the propoſed Number be above 10,000, *which is the Limits of the annexed Table*, then the Logarithm of that Number muſt be found, by the Help of the *common Difference* of the Logarithms, which is the laſt Column of every Page under *Diff.* Thus: *Find*

Find the Logarithms of the firſt four Figures of the given Number, without its Index, as above; and multiply the common Difference which ſtands againſt the Logarithm, under Diff. *with the other Figures of the given Number, caſting off ſo many Figures of that Product as there are in the Multiplicator; then add the remaining Figures of that Product to the Logarithm of the firſt four Figures, and to their Sum prefix the proper Index; and you will have the compleat Logarithm required.*

Suppoſe it were required to find the Logarithm of 698,476. Firſt, the Logarithm of 6,984 is found in the Table, as above, to be 844,104, and againſt it, under *Diff.* is 62. This 62 being multiplied with 76, the other two Figures of the given Number, produces 4,712. Cut off 12, *viz.* the two laſt Figures, and then add 47 to the Logarithm laſt found, and the Sum will be 844,151; to which prefixing 5, the proper Index of the given Number 698,476, it will be 5.844151, the Logarithm required.

PROB. XIII.

To find the Number to any given Logarithm.

Omit the Index of the given Logarithm, and then ſeek it in the Table of Logarithms, and if exactly found there, then the Number in the firſt Column under *Num.* with that on the Top over Logarithm, will be the Number required. But if the given Logarithm, without its Index, cannot be exactly found in the Table, then the proper Number agreeing to that Logarithm may be found by the Help of the common Difference of Logarithms: Thus:

From the given Logarithm ſubtract the next leſs, and to the Remainder annex Cyphers; then divide it by the common Difference found againſt the next Logarithm, under Diff. *and the Quotient, will be a Number that muſt be annexed to the Number already found againſt the next leſs Logarithm, according as the Index of the given Logarithm denotes.*

Suppoſe, 5.660279 were a given Logarithm, and it were required to find the natural Number anſwering it.

The Number ſought muſt conſiſt of 6 Places of
igures in whole Numbers, as appears by its Index
; which being omitted, ſeek in the Table of Loga-
thms for 660279; but not finding it exactly there,
ke the next leſs to it, *viz.* 660201 ſtanding under
, and againſt 547: Therefore, the firſt four Fi-
ures of the Number ſought muſt be 4,573, and
e common Difference found againſt 660201, un-
er *Diff.* is 95.

Then for the Logarithm 660279
Subtract the next leſs, *viz.* 660201

Remains 78

To which annex two Cyphers, becauſe there is
et wanting two Places of Figures, and it will be
,800, which being divided by the common Diffe-
ence 95, the Quotient will be 82, which muſt be
nnexed to 4,573, and the Sum will be 457,382, the
Number anſwering to the given Logarithm 5.660279.
hus the Logarithm of any given Number may be
aſily found altho' it exceeds the Limits of the Table
y 1,203 Places of Figures, and alſo the Number
greeing to any given Logarithm, without the Help
f ſuch a Table of proportional Parts, as is uſually
nſerted along with the Table of Logarithms for
at Purpoſe.

PROB. XIV.

Having given a ſhort Deſcription of Sines, Tan-
ents, *&c.* we ſhall here ſhew the Geometrical Con-
ruction of thoſe and other Scales commonly uſed
n projecting the Sphere in Plano and in Trigono-
etry, Navigation, Dialling, and other Parts of
ractical Mathematicks, as they are deduced from a
ircle.

Upon a Sheet of fine Paſte-board, or ſuch like *Fig.* 6.
Matter, deſcribe a Circle A B D C with any Radius,
hich croſs at right Angles with the Diameters A B
nd C D; then continue A D to G, and upon the
oint B raiſe B F perpendicular to C B. Draw the
Chord A B, and divide the Quadrant A B into 9 equal
Parts,

Parts, ſetting the Figures 10, 20, 30, &c. to 9 each of which 9 Parts again ſubdivide into 10 eq Parts, and then the Quadrant will be divided in 90 Degrees. Set one Foot of the Compaſſes in t Point A, transfer the ſaid Diviſions to the Cho Line AB, and ſet thereto the Figures 10, 20,30, & and the Line of Chords A B will be divided.

To project the *Sines*, divide the Arch BD in 90 Degrees; from each of which Degrees, let f Perpendiculars on the Semidiameter EB, which Pe pendiculars will divide EB into a Line of Sines, which ſet the Numbers 10, 20, &c.

To project the *Line of Tangents*, from the Cent E, and thro' every Diviſion of the Arch BD, dra the right Lines cutting BF, which will divide it i to a Line of Tangents, ſet thereto the Numb 10, 20, &c.

To project the *Line of Secants*, transfer the D ſtances E 10, E 20, &c. From the Tangent Li upon the Line EG, and ſet thereto the Numb 10, 20, &c. The Line EG will be divided in Line of Secants.

To project the *Semitangents*, draw Lines from t Point C thro' every Degree of the Quadrant A and they will divide the Semidiameter AE into Line of Semitangents; but becauſe the Semita gents on Scales run to 160 Degrees, continue t Line AE, and draw Lines from the Point C thr the Degrees of the Quadrant CA, cutting AE, a you will have the Line of Semitangents to 160 De grees, &c.

To project the *Rhumb-line*, from every eighth Pa of the Quadrant AC, ſet one Foot of the Compaſ in A, deſcribe an Arch cutting the Chord AC, whi will divide AC into a Line of whole Rhumbs.

To project the *Line of Longitude*, draw HD equ and parallel to the Radius CE, which divide i to 60 equal Parts, every 10 of which numb Now from every one of theſe Parts let fall Perpend culars to CE, cutting the Arch CD; and havin drawn the Chord CD, with one Foot of the Com paſſes in D, transfer the Diſtances from D to eac the Points in the Arch C D on the Chord C D, d ſet thereto the Numbers 10, 20, *&c.* and the ne of Longitude will be divided.

Theſe are all the Lines commonly put upon one de of the Plain Scale, except equal Parts, which nt no Deſcription: And on the other Side is a De- nal or Diagonal Scale, on which an Inch, or ſome rt thereof, as $\frac{1}{2}$ or $\frac{1}{4}$ is divided into 100 equal rts, by Diagonals.

f the Uſes of the Chords, Sines and Tangents, &c. *upon the Rule.*

The Chords are to lay off the Quantity of an An- e deſired upon a given Point in a right Line, and measure the Quantity of an Angle already laid own. The firſt is done, by taking the Extent of Degrees of Chords between the Compaſſes, and eſcribing an Arch about the angular Point; then ying off the Number of Degrees propoſed upon e ſaid Arch, and drawing a right Line from the angular Point. And the latter, by making an Arch of Degrees of Chords about the angular Points, d then taking the Chord of the ſaid Arch, be- een the Compaſſes, and meaſuring it on the Line f Chords.

Example, To make an Angle of 30 Degrees on the Point A. Take 60 Degrees of Chords in the Com- aſſes, and ſetting one Foot in A deſcribe the Arch DC; then take off 30 Degrees from the Chords, lay hem off from D to C, and draw the Line A C. The ngle C A B will be 30 Degrees. *Fig.* 7.

To meaſure an Angle, ſuppoſe C A B. Take 60 Degrees of Chords, between the Compaſſes, and caſt- ng one Foot in A, deſcribe the Arch C D; then ake the Diſtance from C to D, which, meaſured on he Chords, will reach to 30 Degrees, the Quan- ity of the Angle ſought.

The Sines are to project the Sphere orthogra- hically, *&c.*

The Tangents, Half-Tangents and Secants, are uſed in finding the Centers and Poles of projected Circles in the Stereographical Projection of the Sphere, *&c.*

The Rhumbs, are to lay down the Angles of Ship's Way in Navigation.

And the Line of Longitude determines, by Inſpection, how many Miles there are in a Degre of Longitude, in each ſeveral Latitude : As, i the Latitude of no Degrees, that is, under th Equator, 60 Miles make a Degree; in the Latitud of 40 Degrees, 46 Miles make a Degree; in the Latitude of 60 Degrees, 30 Miles make a Degree; i the Latitude of 80 Degrees, 10 Miles make a Degree.

Having thus laid the Foundation, we ſhall next ſhew the Reſolution of all right lined Triangles in as plain and familiar Method as poſſible.

SECT. II.

Of reſolving TRIANGLES.

THEOREM I.

Fig. 8.

IN any right angled Triangle, if either of the Legs be made Radius, the other Leg will be the Tangent of its oppoſite Angle, *e. g.* If A D be made Radius, B D will be Tangent of the Arch d D = B A D; and if B D be made Radius, A D will be Tangent of the Angle B. But if the Hypothenuſe A B be made Radius, the Legs B D and D A will be the Sines of their oppoſite Angles A and B.

THEOREM II.

Fig. 9.

The Sides of every right-lined Triangle are in Proportion to one another, as the Sines of their oppoſite Angle.

In the Triangle A B C make A F = B C, and let fall the Perpendiculars from F and B to the Side A C, deſcribe the Arches H B and F I. Then B D will be the Sine of the Angle at C, and F E the Sine of the Angle at A, and the Triangles A B D and A F E are ſimilar. Therefore A F : A B : : F E : B D, or as A B : B D : : A F : F E, and A F being equal to B C, and the two Perpendiculars being Sines, it will be, as the Side B C : Side B A : : Sine of the Angle A : Sine of the Angle C; as the Side A B : Sine of the Angle C :: Side B C : Sine of the Angle A.

THEOREM III.

As the Sum of the Legs about an Angle, is to their Difference; ſo is the Tangent of half the Sum of the oppoſite Angles, to the Tangent of half their Difference. *Fig.* 10.

Produce, in the Triangle C F D, the Side F D, and make B F = C F; then B D will be the Sum of the Legs, and G D half the Sum; if you take from which the Leg F D, the Remainder G F = $\frac{1}{2}$ the Difference of the Legs; draw C B, and biſect it in A, and draw A F, which will be perpendicular to it, and the Angle C A F = the Angle B F A, (by Prop. 8. *Euclid*, l. 1.) but the Angle C F B = the Angle F C D + the Angle D: (by Prop. 32. *Euclid. l.* 1.) Therefore the Angle C F A = $\frac{1}{2}$ the Sum of the Angles F C D + D; draw A G, which will be parallel to C D, becauſe the Sides C B and B D are biſected in A and G; then draw E F parallel to C D, which will be parallel to A G; the Angle C F E = the Alternate Angle F C D, the leſſer Angle of the Triangle, becauſe the Angle C F B = the Angles C + D and E F B = the Angle D, take from both the Angle D, then C F F = the Angle C, which taken from the Angle C F A = $\frac{1}{2}$ the Sum of the oppoſite Angles, leaves E F A = $\frac{1}{2}$ the Difference of the oppoſite Angles. Now make A F Radius of a Circle, then E A is the Tangent of half the Difference, and A C the Tangent of half the Sum of the oppoſite Angles: and the Triangles B A G, B E F and B C D are ſimilar, (by Prop. 2. *Euclid*. l. 6.) and conſequently the Sides are proportional. Therefore,

B G : G D :: B A : A C

B G : G F :: B A : A E

Therefore, As G D, half the Sum of the Sides, is to G F half their Difference; ſo is A C, the Tangent of half the Sum of the oppoſite Angles, to A E the Tangent of half their Difference.

But the Wholes are as their Halves: Therefor the Sum of the Sides is to their Difference, as the Tangent of half the Sum of the oppoſite Angles, is to the Tangent of half their Difference.

THEOREM IV.

Fig. 11. *In any Triangle whatſoever, as* ACB, *the Squar of the Side* AB *oppoſite to an acute Angle* C, *is exceeded by the Squares of the other Sides,* AC *and* CB *by the Rectangle* BCF *twice taken; which Rectangle is contained under* BC, *one of the Sides comprehending the acute Angle* C *and the Line* FC, *intercepted between the Perpendicular* AF, *let fall upon the Side* BC *from its oppoſite Angle* A, *and the acute Angle* C.

The Square of BC = 2 Rectangles BFC and $FC^2 + FB^2$. And $AC^2 = CF^2 + FA^2$ (by Prop. 47. *Euclid. l.* 1) Wherefore, $BC^2 + AC^2 = 2BFC + BF^2 + 2FC^2 + AF^2$. But $2BFC + 2FC^2 = 2BCF$. Therefore this being ſubſtituted for them; $BC^2 + AC^2 = 2BCF + BF^2 + AF^2$. But $AF^2 + BF^2 = AB^2$ (by Prop. 47. *Euclid. l.* 1.) Therefore this being ſubſtituted for them, $BC^2 + AC^2 = 2BCF + AB^2$. That is, $BC^2 + AC^2$ exceed AB^2 by 2BCF.

Fig. 12. The Theorem is true, although the Perpendicular fall without the Triangle. And the Demonſtration is almoſt the ſame. For $AC^2 = BA^2 + CB^2 + 2CBF$. Add on both Sides, CB^2, then $AC^2 + CB^2 = AB^2 + CB^2 + 2CBF = AB^2 + 2BCF$.

From this Theorem, and the 47th Prop. Euclid l. 1 *we have the Meaſure of any Triangle whatſoever, whoſ three Sides are known, although the Area be altogethe inacceſſible. For by the Help of theſe Theorems the Per pendicular is known, although the Impediments of th Place ſhould not allow us to mark it out. But note*

Tha

That the Perpendicular, multiplied by half the Side on which it falls, produces the Area of the Triangle.

Let there be any Triangle, as ABC, *having its Sides known. It is required, to find the Perpendicular* AF *falling from the Angle* A *upon the oppoſite Side* BC. *Take the Square of the Side* AB, *oppoſite to the acute Angle* C, *out of the Sum of the Squares* AC *and* BC: *By the laſt Theorem, the Remainder ſhall be the Rectangle* BCF *twice taken. Divide half of the Remainder, that is, the Rectangle* BCF *by the known Side* BC; *thence will ariſe the right Line* CF. *Take the Square of the Line* CF, *out of the Square of* AC: *The Remainder will give the Square of* AF, *whoſe ſquare Root will give the Perpendicular* AF. Fig. 11, or 12.

CASE I.

The two acute Angles B *and* C, *and the Baſe* BA, *being given, to find the Perpendicular* CA. Fig. 12.

1. By making the Hypothenuſe BC Radius.

As the Sine of the Angle C at the Perpendicular 56° 15′	9.9198464
Is to the Baſe AB 121.394	2.0841992
So is the Sine of the Angle B at the Baſe 33° 45′	9.7447390
	11.8289382
To the Perpendicular AC 81.113	1.9090918

2. By making the Baſe AB Radius. Fig. 13.

As Radius AB 45°	10.0000000
Is to the Baſe BA 121.394	2.0841992
So is the Tangent of the Angle B 33° 45′	9.8248926
To the Perpendicular AC 81.113	1.9090918

Fig. 14. 3. By making the Perpendicular A C Radius.

As the Tangent B A of the Angle C at the Perpendicular 56° 15′	10.1751074
Is to the Radius A C 45°	10.0000000
So is the Baſe AB 121.394	2.0821992
To the Perpendicular A C 81.113	1.9090918

In making the Proportions for finding the Sides or Angles of a plain Triangle, it muſt be obſerved, that every Side of a plain Triangle has two Names, and that each Side has one of thoſe Names fixed, *viz.* the *Hypothenuſe*, the *Perpendicular* and *Baſe*. The other Names are precarious according to the Side made Radius, and are called the Words on the ſeveral Sides: Thus, when the Hypothenuſe is made Radius, then the Word or the Hypothenuſe is Radius, and the Word on the Baſe is the Sine of its oppoſite Angle; as alſo, the Word on the Perpendicular is the Sine of its oppoſite Angle; but when the Perpendicular is made Radius, then the Word on the Baſe is the Tangent of its oppoſite Angle, and the Word on the Hypothenuſe is the Secant of the ſame Angle; and when the Baſe is made Radius, then the Word on the Perpendicular is the Tangent of its oppoſite Angle, and the Word on the Hypothenuſe is the Secant of that Angle. Theſe Things being obſerved, the Way to form a Proportion to find the Side of a Triangle, is thus:

Firſt, Suppoſe one Side of a Triangle to be made Radius, and obſerve, as above, the Word on the ſeveral Sides, the Proportion will be,

As the Word on the Side given,
Is to the given Side;
So is the Word on the Side required,
To the Side required.

Thus we ſee that what is ſought muſt always ſtand in the fourth or laſt Place, and therefore, ſince the Perpendicular is ſought, that muſt be the laſt of the four Terms; place it then with the Point of Interrogation after it to ſhew it is required.

In the Rule of Proportion, the ſecond and fourth

Term

Terms being always of the ſame Nature, and the Perpendicular being a Length ſought, and the Baſe the only Length given, the Baſe therefore muſt be in the ſecond Place, and is to be wrote with four Points after it thus : : to ſhew that the Proportion disjoins there.

Again, we are here to obſerve, that the Nature of Logarithms, or their Proportion to one another, is ſuch, that Addition ſerves inſtead of Multiplication, and Subtraction for Diviſion; therefore the Logarithms of the two laſt Terms being added together, and from the Sum, the Logarithm of the firſt Term being ſubtracted, the Remainder 1.9090918 will be the Logarithm of the fourth Term, and the Number anſwering to that Logarithm is 81.113, which is the Perpendicular A C required.

But where Radius is not in Proportion, it may be more readily done by Addition only; for, if inſtead of the firſt Term you ſet its arithmetical Complement, that is, to write down what each Figure wants of 9; thus the arithmetical Complement of 9 9198464, the firſt Term is 0.0801536, which is the ſame as ſubtracting it from 10, then add all the three Terms together, the Sum, abating the Radius, ſhall anſwer the Queſtion

2. To do the ſame by Scale and Compaſſes. Always extend the Compaſſes from the firſt Term to the Term that is of the ſame Kind, whether it be the ſecond or third, that Extent will reach from the remaining Term to the Anſwer. Thus, in the firſt Proportion, extend the Compaſſes from 56°15′ to 33°45′ in the Line of Sines; that Extent will reach in the Line of Numbers from 121.39 to 81 11 the Anſwer. In the ſecond Proportion, extend the Compaſſes from 45° to 33°45′ in the Line of Tangents; that Extent will reach from 121.39 to 81.11 in the Line of Numbers. In ſome Caſes it may be needful to uſe Croſs-work, that is, to extend from the firſt Term in the Line of Sines to the ſecond in the Line of Numbers, or from the firſt Term in Tangents to the ſecond in Numbers, &c: but in moſt Caſes it is better to work by the Directions

above, except when the Extent is too large for the Compaſſes.

3. By the Sliding Rule. Suppoſe, the Line of Sines on the Rule to be marked with SS, and the Line of Sines on the Slider with S. Then the firſt Proportion will be thus wrought. Set 33°45′ on S to 56°15′ on SS; then againſt 121.39 on A, is 81.11 on B. (A ſignifies the double Line of Numbers upon the Rule, and B the double Number on the Slider). The ſecond Proportion may be thus wrought: Set 33°45′ in the Tangents to Radius; then againſt 121.39 on A, is 81.11 on B. Or, if the Slider be ſo turned as the Tangents and double Numbers may ſlide one by another, then the Radius may be ſet, *viz.* 45° of Tangents to 121.39 in the Line of Numbers; then againſt 33°45′ in Tangents is 81.11 in the Line of Numbers. The third Proportion may be wrought as this laſt.

Fig. 15. 4. To do the ſame geometrically. Draw the Baſe BA, and from a Diagonal Scale, or Scale of equal Parts, take with the Compaſſes 121.39, and ſet from B to A, and upon A raiſe a Perpendicular; then take 60 Degrees from the Line of Chords with the Compaſſes, and ſet one Foot in B, deſcribe the Arch DE, and from the ſame Line of Chords 33°45′ and ſet from D to E; and draw the Line DC till it cut the Perpendicular in C: Then if you meaſure AC by the ſame Scale you took BA from, you will find it 81.11; and if you meaſure BC, you will find it 146. The next Caſe will be ſo reſolved.

CASE II.

Fig. 12. *The two acute Angles* B *and* C, *and the Baſe* BA *being given, to find the Hypothenuſe* BC.

1. By making the Hypothenuſe BC Radius.

As the Sine of the Angle C 56°15′	9.9198464
Is to the Baſe BA 121.394	2.0841992
So is the Radius 90°	10.0000000
To the Hypothenuſe BC 146	2.1643528

2. By making the Baſe A B Radius.

Radius B A	10.0000000	*Fig.* 13.
to the Baſe B A 121.394	2.0841992	
is the Secant of the Angle B 33°45′	10.0801536	
the Hypothenuſe B C 146	2.1643528	

3. By making the Perpendicular A C Radius.

the Tangent of the Angle C 56°15′ Arith. Compl.	9.8248926	*Fig.* 14.
to the Baſe B A. 121.394	2.0841992	
is the Secant of the Angle C	10.2552610	
o the Hypothenuſe B C 146	2.1643528	

By Scale and Compaſſes.

To work the firſt Proportion extend the Compaſſes om the Sine of 56°15 to 90°, which Extent ill reach from 121.39 to 146 in the Line of Num- ers.

By the Sliding Rule.

Set 56°15′ upon S to 90° upon S S, then againſt 21.39 upon B, is 146 upon A. But if you turn the lider, ſo as the Sines and double Numbers may ſlide ne by another, you may ſet 56° 15′ of Sines to 21. 39 in the Line of Numbers: Then againſt 90° f Sines you will have 146 in the Line of Numbers, nd will find 33° 45 of Sines to be againſt 81. 11 of Numbers. So that you may obſerve, if 90 of Sines e ſet to the longeſt Side or Hypothenuſe, you will nd every Angle againſt its oppoſite Side.

CASE III.

wo acute Angles B *and* C, *and the Hypothenuſe* B C *being given, to find the Baſe* B A. *Fig.* 12.

1. By making the Hypothenuſe B C Radius.

s Radius 90°	10.0000000
to the Hypothenuſe B C 146	2.1643528
o is the Sine of the Angle C 56°15′	9.9198464
To the Baſe B A 121. 394.	2.0841992

2. By making the Baſe B A Radius.

Fig. 13. As the Secant of the Angle B 33° 45 10.0801536
Is to the Hypothenuſe B C 146 2.1643528
So is the Radius 90° 10.0000000

To the Baſe B A 121.394 2.0841992

3. By making the Perpendicular A C Radius

Fig. 14. As the Secant of the Angle C 56° 15' Arith Compl. } 9.7447390
Is to the Hypothenuſe B C 146 2.1643528
So is the Tangent of the Angle C 56° 15' 10.1751074

To the Baſe B A 121.394 2.0841992

By Scale and Compaſſes.

To work the firſt Proportion, extend the Compaſſes from 90° to 56° 15 in the Sines, which Extent will reach from 146 to 121.394 in the Line of Numbers.

By the Sliding Rule.

Set 56° 15' upon S to 90° upon S S; then againſt 146 upon A, is 121.394 upon B.

By Geometrical Protraction.

Fig 15. Draw the Line B C, and from the Scale of equal Parts take 146, and ſet from B to C, with 60 Degrees of Chords deſcribe the Arch D E; and ſet 56° 15' from E to D, and draw C A, and from B draw B A perpendicular to A C; then when A B is meaſur'd the Scale, it will be found to contain 121.39.

CASE IV.

Fig. 13. *The Baſe* B A *and the Perpendicular* C A *being given, to find the two acute Angles* B *and* C.

1. By making the Baſe B A Radius.

As the Baſe B A 121.394 2.0841992
Is to the Radius 45° 10.0000000
So is the Perpendicular A C 81.113 1.9090918

To the Tangent A C of the Angle B 33° 45 } 9 8248926
Whoſe Complement is 56° 15' the Angle C.

2. By making the Perpendicular A C Radius.

Fig. 14.

As the Perpendicular A C 81.113	1.9090918
Is to Radius 45° 15′	10.0000000
So is the Baſe 121.394	2.0841992
To the Tangent of the Angle C 56° 15′	10.1751074

Whoſe Complement is 33° 45′ the Angle B.

By Scale and Compaſſes.

To work the firſt Proportion, extend the Compaſſes from 121.394 to 81.113 in the Line of Numbers, that Extent will reach from 45° to 33° 45′ in the Tangents. For the ſecond Proportion, extend from 18 113 to 121.394 in the Line of Numbers, that Extent will reach from 45° to 56° 15′ in the Tangents.

By the Sliding Rule.

Let the Tangents and Numbers ſlide together, then ſet 121.394 in the Line of Numbers to 45° of Tangents, and againſt 81.113 in the Line of Numbers is 33° 45′ and its Complement 56° 15′ in the Line of Tangents.

This Caſe is done Geometrically, as the ſixth.

CASE V.

The Baſe B A *and the Hypothenuſe* B C *being given, to find the two acute Angles* B *and* C.

Fig. 12.

1. By making the Hypothenuſe B C Radius.

As the Hypothenuſe B C 146	2.1643528
Is to the Radius 90°	10.0000000
So is the Baſe B A 121.394	2.0841991
To the Sine of the Angle C 56° 15′	9.9198463

Whoſe Complement 33° 45 is the Angle B.

2. By making the Baſe Radius.

As the Baſe B A 121.394	2.0841992
Is to the Radius 90°	10.0000000
So is Hypothenuſe 146	2.1643528
To the Secant of the Angle B 33° 45′	10.0801536

By Scale and Compaſſes.

For the firſt, extend the Compaſſes from 146 to 121.394 in the Line of Numbers, which will reach from 90° to 56° 15′ in the Sines.

By the Sliding Rule.

Set 146 upon A to 121.394 upon B, then againſt Radius on S S, is 56° 15′ on S.

By Geometrical Protraction.

Fig. 15. Draw the Baſe B A, and from a Scale of equal Parts take 121.394 and ſet from B to A, and upon A raiſe the Perpendicular A C; then from the ſame Scale of equal Parts take 146, and ſet one Foot of the Compaſſes in B with the other croſs A C in C, and draw B C. Then with 60° of Chords deſcribe the Arch D E, and meaſure D E with the Compaſſes on the Line of Chords, which will be 33° 45′ the Meaſure of the Angle B, whoſe Complement is the Angle C.

CASE VI.

Fig. 16. *The Baſe* B A *and the Perpendicular* C A *being given, to find the Hypothenuſe* B C

In this, and the next Caſe, we are firſt to find the acute Angle, and from thence the third Side.

1. The Perpendicular A C being made Radius.

As the Perpendicular C A 81.113	1.9090918
Is to the Baſe B A 121.394	2.0841992
So is the Radius 45°	10.0000000
To the Tangent of the Angle C 56° 15′	2.1643528

2. The Hypothenuſe B C being made Radius.

As the Sine of the Angle C 56° 15′	9.9198464
Is to the Baſe B A 121.394	2.0841999
So is the Radius 90°	10.0000000
To the Hypothenuſe 146	2.1643528

By

By Scale and Compaſſes.

Extend the Compaſſes from 81.113 to 121.39 n the Line of Numbers, that Extent will reach from 45° to 56° 15′ in the Sine of Tangents; then extend from 56° 15′ to 90° in the Sines, that Extent will reach from 121.394 to 146.

By the Sliding Rule.

Set 81.113 in the Line of Numbers to 45° in the Tangents; then againſt 121.39 in the Numbers is 56° 15′ in the Tangents; then ſet 56° 15′ on S to 90° on SS; then againſt 121.39 on B is 146 on A.

By Geometrical Protraction.

Draw the Line BA, and from a Scale of equal Parts take 121.394 in the Compaſſes, and ſet from B to A, and upon A erect the Perpendicular AC; and from the ſame Scale take 81.113, and ſet from A to C; then draw the Hypothenuſe BC, take BC in the Compaſſes, and on the Scale it will be found 146. Then with 60° of Chords deſcribe the Arch DE, meaſure DE with the Compaſſes on the Line of Chords, which will be 33° 45′ the Meaſure of the Angle B, whoſe Complement is the Angle C. *Fig.* 15.

CASE. VII.

The Baſe BA, *and the Hypothenuſe being given, to find the Perpendicular* AC. *Fig.* 12.

1. The Hypothenuſe BC being made Radius.

As the Hypothenuſe BC 146	2.1643528
Is to the Radius 90°	10.0000000
So is the Baſe BA 121.394	2 0841992
To the Sine of the Angle C 56° 15′	9.9198464

Whoſe Complement is the Angle B 33° 45′

2. The Hypothenuſe BC being made Radius.

As Radius 90	10.0000000
Is to the Hypothenuſe BC 146	2.1643528
So is the Sine of the Angle B 33° 45′	9.7447390
To the Perpendicular AC 81.113	1.9090918

By

By Scale and Compaſſes.

For the firſt Operation, extend the Compaſſe from 146 to 121.394 in the Line of Numbers, tha Extent will reach from 90° to 56° 15′ in the Sines. Then for the ſecond Operation, extend the Compaſſes from 90° to 33° 45′ in the Sines, that Extent will reach from 146 to 81.113 in the Line o Numbers.

By the Sliding Rule.

For the firſt, ſet 146 on A to 121.394 on B, the againſt 90 on S S there will be found 56° 15′ on S Then for the ſecond, ſet 90° on SS to 33° 45′ o S and againſt 146 on A, is 81.113 on B.

By Geometrical Protraction.

Fig. 15. Draw the Line A B, and from the Scale tak 121.394, which ſet from B to A, and upon A raiſ the Perpendicular A C, then from the Scale tak 146, and ſet one Foot of the Compaſſes B, croſ the Perpendicular in C, and meaſure A C on th Scale, which will be 81.113.

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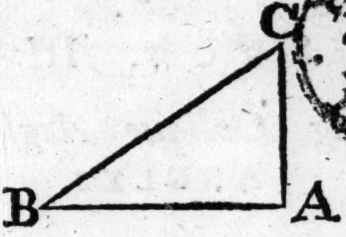


The Seven Caſes of Plain Triangles.

Right Angled.			
Caſes.	Given	Required.	Proportions.
I.	A B and B	A C	1. s C : B A : : s B : A C. 2. R : B A : : t B : A C. 3. t C : B A : : R : A C.
II.	A B and C	B C	1. s C : B A : : R : B C 2. R : B A : : seB : B C 3. t C : B A : : seC : B C
III.	B C and B	B A	1. R : B C : : s C : B A 2. seB : B C : : R : B A 3. scC : B C : : t C : B A
IV.	A B and A C	B and C	1. B A : R : : A C : t B whoſe Complement is C. 2. C A : R : : B A : t C whoſe Complement is B.
V.	B C and A C	B and C	1. B C : R : : B A : s C whoſe Complement is B 2. B A : R : : B C : se B whoſe Complement is C
VI.	A B and A C	B C	1. C A : B A : : R : t C. Then again, 2. s C : B A : : R : B C.
VII.	A B and B C	A C	1. B C : R : : B A : s C whoſe Complement is B Then again, 2. R : B C : : s B : A C.

SECT. III.

Of ſolving Oblique Angled TRIANGLES.

CASE I.

Fig. 17. *The Angles* CAB 62° 30′ *and* CBA 37° 30′ *and the Side* AC 350 *Feet being given, to find the other two Sides* CB *and* AB.

As the Sine of the Angle CBA 37° 30′ Arith. Compl.	0.2155529
Is to the Side AC 350	2.5440680
So is the Sine of the Angle CAB 62° 30′	9.9479289
To the Side BC 509.976.	2.7075498

For the Side AB.

As the Sine of the Angle CBA 37° 30′ Arith. Comp.	0.2155529
Is to the Side AC 350	2.5440680
So is the Sine of the Angle ACB 80°	9.9933515
To the Side AB 566.203	2.7529724

The Parts required in the ſeveral Caſes of oblique Trigonometry, may be found with the Scale and Compaſſes by extending the Compaſſes from the firſt Term of the Proportion, to the ſecond, and the ſame Extent will reach from the third Term to the fourth required. Alſo the Parts required, may be found by the Sliding Rule, by ſetting the firſt Term againſt the ſecond, then oppoſite to the third Term, the fourth Term may be found.

CASE II.

Fig. 17. *The two Sides* AC 350 *and* CB 509.976, *and the Angle* CAB 62° 30′ *oppoſite to one of the given Sides,* CB *being given, to find the Angle* CBA *oppoſite to the other Side* AC.

As

As the Side BC 509.976 Arith. Comp.	7.2924502
Is to the Sine of the Angle A 62° 30′	9.9479289
So is the Side AC 350	2.5440680
To the Sine of the Angle B 37° 30′.	9.7844471

Or, if the two Sides AB and BC and the Angle oppoſite to the Side BC had been given, and the Angle C had been required; then,

As the Side BC 509.976 Arith. Comp.	7.2924502
Is to the Sine of the Angle A 62° 30′	9.9479289
So is the Side AB 566.203	2.7529724
To the Sine of the Angle C 80.	9.9933515

CASE III.

The Side AB 566.203, *the Side* AC 350, *and the Angle* A 62° 30′ *comprehended between the Sides* AB *and* AC *being given, to find the Angles* ACB *and* ABC. *Fig.* 17.

AB=566.203	180°	
AC=350	Subt. 62 30′	
The Sum 916.203	Rem. 117 30=	Sum of the Angles B and C.
Diff. 216.203	58 45=	half the Sum.

As 916.203 the Sum of the Sides Arith. Comp.	7.0380083
Is to 216.203 the Difference of the Sides	2.3348617
So is the Tangent 58° 45′ half the Sum of the oppoſite Angles.	10.2169438
To the Tangent 21° 15′ half the Diff. of the oppoſite Angles, (by *Theor.* 3. *Sect.* 11.)	9.5898138

If 21° 15′ be added to 58° 45′ the Sum will be 80° the Angle ACB; and if 21° 15′ be ſubtracted from 58° 45′, the Remainder will be 37° 30′ the Angle ABC.

CASE IV.

Fig. 17. *The Side* AB 566.203, *the Side* BC 509.976 *and the Angle* B 37° 30' *comprehended between the Sides* AB *and* AC *being given, to find the third Side* AC.

The Side AB	566.203	180° 00'
The Side BC	509.976	37 30
The Sum	1076.179	142° 30'
Difference	56.227	71° 15'

As 1076.179 the Sum of the Sides Arith. Compl.	6.9681154
Is to the Difference 56.227	1.7499449
So is the Tangent of half the Sum of the oppoſite Angles.	10.4692187
To the Tangent 8° 45' half the Difference of the oppoſite Angles, (by *Theor*. 3. *Sect*. II.)	9.1871790

If 8° 45' be added to 71° 15', the Sum will be 80° the greater Angle C, and being ſubtracted the Remainder 62° 30 the leſſer Angle A. Then,

As the Sine of the Angle A 62° 30' Arith. Compl.	0.0520711
Is to the Side BC 509.976	2.7075498
So is the Sine of the Angle B 37° 30'	9.7844471
To the Side AC 350, which is required, (by *Caſe* I. *Sect*. III.)	2.5440680

CASE V.

Fig. 18. *The three Sides* AB 213.5, AC 103.5, *and* BC 250.2 *of an oblique Triangle* ABC *being given to find the three Angles.*

As

As the greateſt Side C B 250.2 Arith. Compl.	7.6017127
Is to the Sum of the other two Sides A B and A C 321.	2.5065050
So is the Difference B E 106 of the two Sides A B and A C.	2.0253059
To the Difference B F 135.995 of the Segments of the Baſe.	2.1335236

Which Difference 135.995, ſubtracted from B C 250.2 the greateſt Side, leaves F C 114.205, the half whereof is G C 57.1025 the leſſer Segment; which if ſubtracted from B C 250.2, the Remainder B G will be 193.0975, the greater Segment of the Baſe. Thus the oblique Triangle is reduced into two right angled Triangles, *viz.* A B G and A G C both right angled at G, in each of which there is given the Hypothenuſe and Baſe: So the Angles may be found by (Caſe V. *Of right angled plain Trigles*;) thus:

As the Hypothenuſe A B 213.5.	2.3293979
Is to the Radius	10.0000000
So is the Baſe B G 193.0975	2.2857762
To the Sine of the Angle B A G 64° 44′ 51″	9.9563783

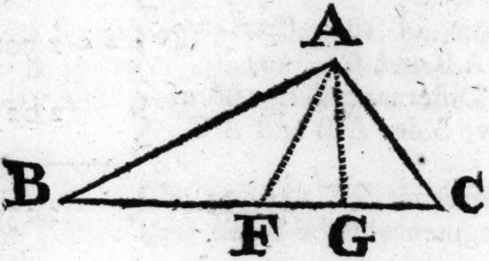
The Complement of B A G is A B G 25° 15′ 9″

Again, in the Triangle A C G.

As the Hypothenuſe A C 107.5	2.0314085
Is to the Radius	10.0000000
So is the Baſe G C 57.1025	1.7566552
To the Sine of the Angle G A C 32° 5′ 8″	9.7252467

The Complement of G A C is A C G 57° 54′ 52″.

This Caſe is demonſtrated from the fourth Theorem of Sect. II.



The Five Caſes of oblique angled Triangles.

Caſes.	Given.	Required.	Proportions.
I.	A C A and B	A B	s B : A C : : s C : A B.
II.	A C C B and B.	A	C A : s B : : C B : s A.
III.	A C C B and C.	A and B	C B + C A : C B − A C : : t ½ A + B : t ½ the Difference, which ½ Dif. $\left\{\begin{matrix}\text{added to}\\ \text{ſub.from}\end{matrix}\right\}$ the ½ Sum gi. the $\left\{\begin{matrix}\text{greater}\\ \text{leſſer}\end{matrix}\right\}$ Angle
IV.	A C C B and C.	A B	Find the Angles A and B by the laſt Caſe; then, by the firſt Caſe, the Side AB will be found.
V.	A B A C & C B.	A, B and C	B C : A B + A C : : A B − A C : B F. Then B C − B F = F C and ½ F C is C G.

SECT. I.

Of Trigonometrical PROBLEMS.

PROB. I.

To meaſure an acceſſible Altitude.

LET AB *repreſent a Tower, Steeple, &c. whoſe Height is required?*

Firſt, with the Quadrant, or other Inſtrument, find the Quantity of the Angle C, which ſuppoſe to be 52° 30′, then meaſure the Diſtance A C; which ſuppoſe to be 85 Feet; then by *Caſe* I. of plain Triangles:

As the Sine of the Angle C B A 37° 30′ Arith. Compl.	0.215553
Is to the Baſe A C 85 Feet	1.929419
So is the Sine of the Angle C 52° 30′	9.899466
To the Altitude A B 110.8	2.044438

Or thus,

As Radius	10.000000
Is to the Baſe A C 85	1.929419
So is the Tangent of the Angle C 52° 30′	10.115019
To the Altitude 110.8	2.044438

Note, *That in this, and all ſuch Caſes, you muſt add the Height of your Eye, or Inſtrument to the Altitude before found.*

PROB. II.

To meaſure an inacceſſible Altitude.

Let A B *be a Church Steeple, whoſe Height is required; but by reaſon of a River, or ſome other Obſtacle, you cannot come to the Foot of it at* A.

Firſt, Take, with the Quadrant at C, the Angle of Altitude, which, ſuppoſe to be 26° 30′; then meaſure in a right Line towards the Steeple to D, which ſuppoſe to be 75 Feet, and at D again obſerve the Angle of Altitude, which, let be 51° 30′.

Now, the two viſual Lines C B and D B, and the meaſured Diſtance C D form the oblique angled Triangle C B D, wherein are given all the Angles, and the Side C D, the Angle B C D being 26° 30′ and the Complement of A D B 51° 30′ to 180 is the obtuſe Angle B D C 128° 30′, and conſequently, the third Angle C B D is 25°. But this Angle may be more readily found by ſubtracting B C D from A D B (by *Euclid.* l. 1. Prop. 32.) Then, by Caſe I. *Of oblique angled plain Triangles,* find the Side B D; Thus:

As the Sine of the Angle C B D 25°30′	0.374052
Is to the Diſtance of C D 75	1.875067
So is the Sine of the Angle C 26° 30′	9.649527
To the viſual Line B D 79.18	1.898640

Then by Caſe I. *Of right angled plain Triangles,*

As Radius	10.000000
Is to B D 79.18	1.989640
So is the Sine of the Angle A D B 51° 32′	9.893544
To the Altitude A B 61.97.	1.729184

PROB. III.

To meaſure the Height of a Steeple, Tower, &c. ſtanding upon a Hill.

Firſt, Find the Angle C A B 44° and the Angle E A B 26°; then meaſure in a ſtreight Line towards the Steeple from A to D 134 Feet: Then again at D, find the Angle C D B 67° 50′, and the Angle E D B 51°. By Caſe I. *Of oblique plain Triangles,* find the viſual Line C D in the Triangle A C D, wherein are given the Angles D A C and A C D and the Side A D; Thus:

As the Sine of the Angle A C D 23° 51′	3.393536
Is to the meaſured Diſtance A D 134	2.127105
So is the Sine of the Angle C A D 44°	9.841771
To the Side C D 230.4	2.362412

Then, as Radius	10.000000
Is to the Side CD 230.4	2.362412
So is the Sine of the Angle CDB 67° 50′	9.966653
To the Side BC 213.3	2.329065

Again, As Radius	10.000000
Is to the Side CD 230.4	2.362412
So is the Sine of the Angle BCD 22° 10′	9.576689
To the Baſe BD 86.92	1.939101

Laſtly,

As Radius	10.000000
Is to the Baſe BD 86.92	1.939101
So is the Tangent of the Angle BDE 51′	10.091631
To the Perpendicular BE 107.3	2.030732

From the whole perpendicular Height BC	213.3
Subtract the perpendicular Height of the Hill BE	107.3
There remains the Height CE of the Steeple	106

PROB. IV.

One Side BC 532 *of an oblique Triangle* ABC, *the Angle* A 110° 30′ *oppoſite to that Side, and the Sum of the other two Sides* AB *and* AC 637 *being given, to find the other two Sides and the Angles ſeverally.* Fig. 22.

Extend the Side BA to D, make AD equal to AC and draw DC, ſo there will be other two oblique angled Triangles BDC and ADC. In the Triangle ABC is given the Angle BAC 110° 30′, in the Triangle ACD is given the Angle CAD 69° 30′ the Complement of the other to 180°; alſo the Triangle ADC is equicrural by Conſtruction, therefore the Angles C and D at the Baſe are equal, and each of them is equal to ½ the given Angle BAC (by Prop. 32. *Euclid.* l. 1.) Now in the Triangle BCD there is given BC 532, BD 637, and the Angle

BDC 55° 15′. Whence the Angle D C B may be found (by Caſe I. *Of oblique Triangles.*)

As the Side of BC 532 Arith. Comp.	7.2740884
Is to the Sine of the Angle BDC 55° 15′	9.9146852
So is the Side B D 637	2.8041394
To the Sine of the Angle BCD 100° 19′	9.9929130

From which ſubtract the Angle A C D 55° 15′, and the Remainder is 45° 4′ for the Angle A C B, and the Angle is 24° 26′, which is found by ſubtracting the Sum of B C D 100° 19′, and D 55° 15′ from 180°. The Sides A B and A C, are found (by Caſe I. *Of oblique Triangles.*)

As the Sign of the Angle B A C 100° 30′ Arith. Compl.	0.0284124
Is to the Side BC 532	2.7259116
So is the Sine of the Angle A C B 45° 4′	9.8499897
To the Side A B 402.08	2.6043187

Again,

As the Sine of the Angle B A C 110° 30 Arith Compl.	0.0284124
Is to the Side B C 532	2.7259116
So is the Sine of the Angle ABC 24° 26′	9.6166164
To the Side AC 234.93 ——	2.3709404

PROB. V.

Fig. 23. *One Side* BC 250.2 *of an oblique Triangle* A B C, *the Angle* B A C 96° 50′ *oppoſite thereto, and the Difference of* BD 106, *of the other two Sides* A B *and* A C *being given, to find the Angles* B *and* C *and the two Sides ſeverally.*

Make AD equal to AC, and draw CD; the Angle DAC being 96° 50′, the Complement thereof to 180° is 83° 10′ for the two Angles ADC and ACD; which being equal one to the other, therefore each of them is half of 83° 10′; and by drawing CD there is alſo another Triangle made, wherein is given BC 250.2 and BD 106, equal to the Difference of the two Sides AB and AC; and there is alſo given the Angle BDC 138° 25′, equal to the Complement of 41° 35′

As the Side BC 250.2 Arith. Compl.	7.6017127
Is to the Sine of the Angle BDC 138° 25′	9.8219775
So is the Side BD 106	2.0253059
To the Sine of the Angle BCD 16° 19′ 52″	9.4489961

To which if ACD 41° 35′ be added, the Sum of the Angle ACB will be 57° 54′ 52″: And if A be added to it, and the Sum ſubtracted from 180, there will remain the Angle ABC 25° 15′ 8″. Find the Sides AB and AC, as in the laſt Problem; Thus:

As the Sine of the Angle BAC 96° 50′ Arith. Compl.	0.0030960
Is to the Side BC 250.2	2.3982873
So is the Sine of the Angle ACB 57° 54′ 52″	9.9280146
To the Side AB 213.5	2.3293979
Again,	
As the Sine of the Angle BAC 96° 50′ Arith. Comp.	0.0030960
Is to the Side BC 250.2	2.3982873
So is the Sine of the Angle ABC 25° 15′ 8″	9.6300247
To the Side AC 107.5	2.0314080

PROB. VI.

In the right angled Triangle ABC *there are given, Baſe* AB 40, *and the Sum of the Perpendicular* AC, *and Hypothenuſe* AD 200, *to find the Perpendicular* AC *and Hypothenuſe* CB *ſeverally.* Fig. 24.

In the Triangle ABD, is given the Baſe and Perpendicular, to find the Angles ABD and ADB; Thus:

As the Baſe AB 40	1.6020600
Is to the Radius 45	10.0000000
So is the Perpendicular AD 200	2.3010300
To the Tangent of the Angle ABD 78° 41′ 24″	10.6989700

Whoſe Complement is the Angle D 11° 18′ 36″, and becauſe CD is equal to CB, therefore the Angle CBD is alſo 11° 18′ 36″; then if 11° 18′ 36″ be ſubtracted from the whole ABD 78° 41′ 24″, there will remain the Angle ABC 67° 22′ 48″. Then to find the Sides.

As Radius	10.0000000
Is to the Baſe	1.6020600
So is the Tangent of the Angle ABC 67° 22′ 48″	10.3802083
To the Perpendicular AC 96	1.9822683

Then ſubtract 96 from 200, and there will remain the Hypothenuſe BC 104.

PROB. VII.

Fig. 25. *Let* B, E *and* D *be three Objects, whoſe Diſtances ar* *known, and* C *a Station from which all the Objects may be ſeen, and the Angles with each Object may be found. What is the Diſtance of each Object?*

BD 106, BE 53, 25 DE 65, 5 *the Angles* BCE 13° 30′ *and* DCE 29° 50′ *are given, to fin* BC, EC *and* DC.

The three Angles of the Triangle BDE, *ar* *found by* Caſe V. *Of oblique angled Triangles.*

Thro' the three Points B, D and C deſcribe Circle, draw the Lines BC, DC and EC, whic laſt continue to F, where it cuts the Circle, dra BF and DF. Then 180— Angle BCD = Angl BFD = 136° 40′ (by Prop. 32. *Euclid*. l. 1.) an the Angle BDF = Angle BCF (by Prop. 21. *Eu* *clid*. l. 3.) and ſo the Angle DBF = Angle DC In the Triangle BFD all the Angles are given, an the Side BD, to find BF = 36, 059 to DF = 7
84

Caſe I. *Of oblique angled Triangles.*) Then ∴ Angle FDB Angle BDE = FDE; then the Triangle FDE, are given the Sides ED, FD and the included Angle FDE, to find the Angle FED = 84° 30′ 24″ and 180 — Angle FED = 95° 29′ 36″; then in the Triangle EDC all the Angles, and the Side DE are given, to find EC = 107, 42 and DC = 131, 05. Laſtly, In the Triangle BDC all the Angles and the two Sides BD and DC are given, to find the third Side BC 151, 3.

PROB. VIII.

Suppoſe, B *and* D *two Stations, whoſe Diſtance is* 47.5 *Yards, from whence the two Objects* C *and* E, *may may be ſeen, and their Angles found by Obſervation,* viz. CBE 49°, EBD 38°, CDB 32°, *and* CDE 56° *it is required to find* BC, BE, DC, DE *and* CE. *Fig.* 26.

In the Triangle CBD, the Side BD and all the Angles are given, to find BC = 28, 7795, and DC = 54,2349, (which is done by Caſe I. *Of oblique angled Triangles*) and in the Triangle BED are given the ſame Things, to find BE = 58,6775 and DE = 36,1475, (by the ſame Caſe.) Laſtly, In the Triangle BCE are given BC, BE and the included Angle CBE to find CE = 45,3378, (by Caſe IV. *Of Oblique angled Triangles.*

PROB. IX.

Let C *and* E *be two Objects, whoſe Diſtance is known, and let* B *and* D *repreſent two Stations, from whence they may be both ſeen, and the horizontal Angles* CBE *and* DBE, CDE *and* CDB *found by Obſervation; but the Diſtance of the two Stations cannot be meaſured, What is the Diſtance of the two Stations, and the Objects from each Stations?* *Fig.* 27.

Draw another Figure cedb, whoſe Side db, ſuppoſe to be any Number, as 10, and ſimilar to

CEDB; now, upon Suppoſition, that bd 10, and the ſmall Figure ſimilar to the great on we can, by the laſt Problem, find $de=7{,}61$, $be=12{,}353116$, $bc=6{,}05885$, $dc=11{,}41787$ and $ce=9{,}5448$. Now, if by working in this Manner, ce had been found $=CE=45{,}3378$, then it is plain, all the Sides had been exactly found; but as it has not been found by ſimilar Triangles, the true Sides may be found by thoſe already diſcovered, thus, as, $ce:CE::cb:CB=28{,}7795$; and as, $ce:CE::cd:CD$; and $cb:CB::bd:BD=47{,}5$; and as, $ce:CE::ed:ED=36{,}147$; and ſo of the reſt.

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